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AN INTRODUCTION TO THE SONAR EQUATIONS WITH APPLICATIONS (REVIS--ETC(U)

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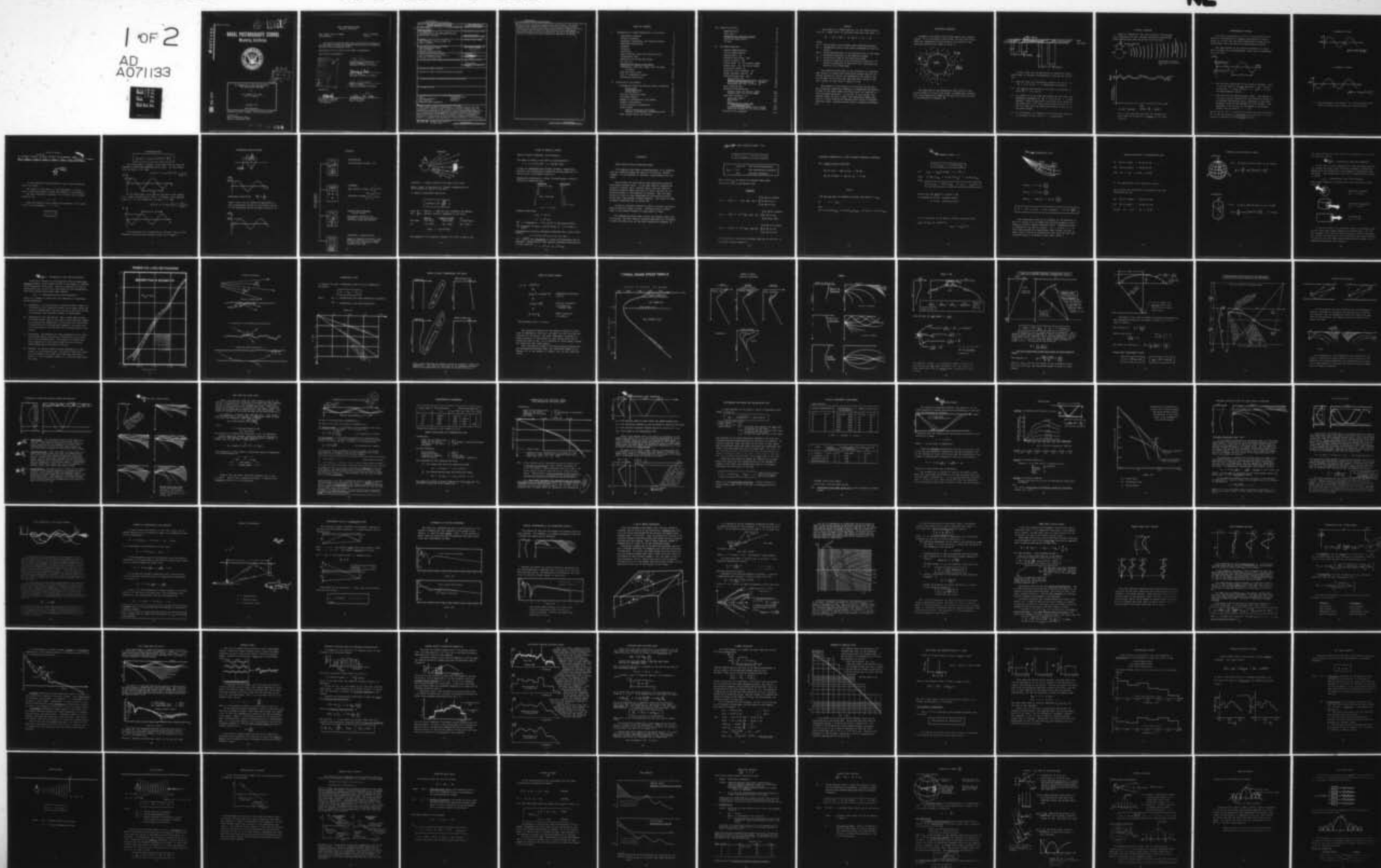

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NAVAL POSTGRADUATE SCHOOL

Monterey, California

AN INTRODUCTION TO THE SONAR EQUATIONS
WITH APPLICATIONS (REVISED)

by

A.B. Coppens, H.A. Dahl,
and J.V. Sanders

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This report provides an introduction to the SONAR equations for those interested in underwater sound as applied to ASW but lacking either the mathematical background or the time for a more rigorous presentation. This is a substantially revised extension of material developed in a previous report (NPS-61Sd-76-071). While this material should be supplemented by lectures or a study guide, we have attempted to design the material so that it is reasonably self-explanatory, communicating many of the essentials

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concepts without requiring extensive verbal amplification. The unusual format has been deliberately chosen to facilitate these goals, and our experiences in presenting these materials have seemed to justify this choice. It is assumed that the reader has some familiarity with trigonometric functions and either has or will develop with the aid of the appendix the facility of handling scientific notation and logarithmic operations.

TABLE OF CONTENTS

PURPOSE

One example of a SONAR equation, for the passive detection of a target which is radiating sound into the ocean, is

$$SL - TL \geqslant NSL + 10 \log w - DI + DT$$

where

SL = Source Level of the target being detected passively

TL = Transmission Loss as the signal propagates to the detector

NSL = Noise Spectrum Level of the ambient noise in the ocean

w = Frequency Bandwidth of the detecting system

DI = Directivity Index of the detecting system

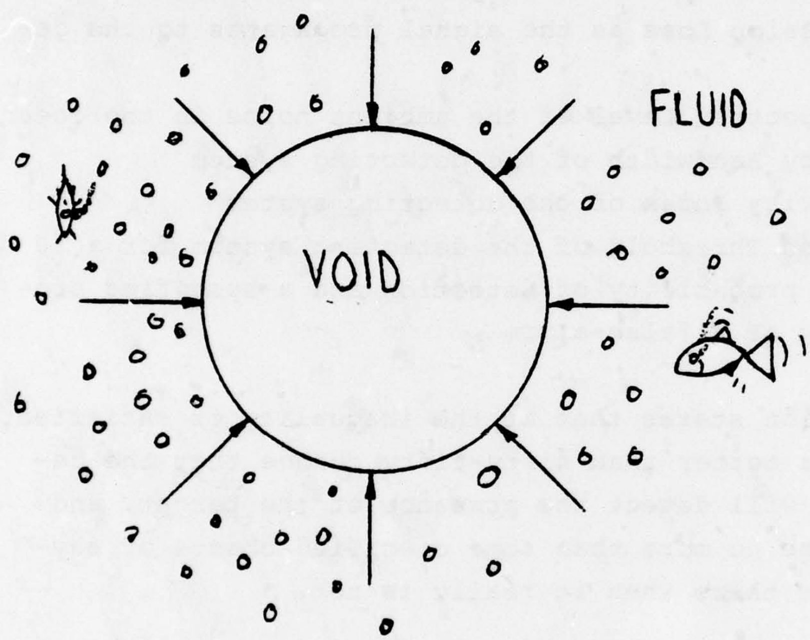
DT = Detection Threshold of the detecting system for a 50 percent probability of detection and a specified probability of a false-alarm

The equation states that if the inequality is satisfied, then there is a better than fifty-fifty chance that the detecting system will detect the presence of the target, and at the same time no more than some specified chance of saying a target is there when it really is not.

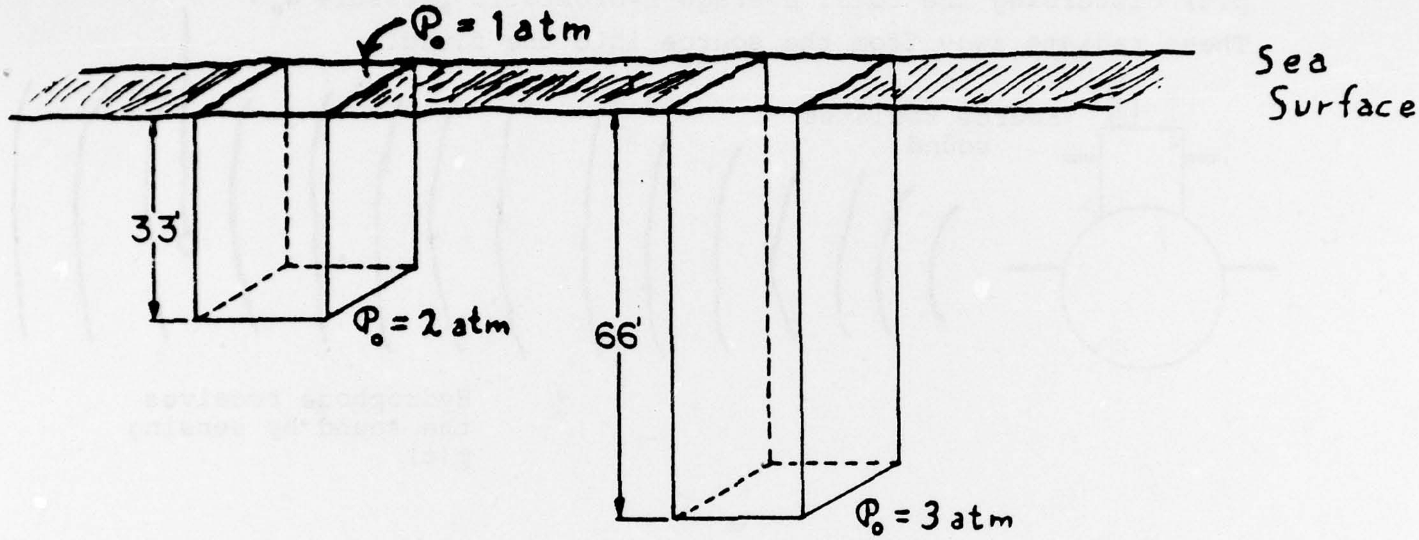
The purpose of this course is to provide the physical and conceptual background necessary to understand the meaning of each of the above terms and others which are combined to form the SONAR equations. At the end of this course, the student should be able to apply the appropriate SONAR equation to a given problem and estimate such things as maximum detection range, optimum receiver depth, necessary bandwidth, etc.

HYDROSTATIC PRESSURE

Suppose a very small hollow glass sphere has a vacuum inside. If it is placed in still water (or any other fluid) there are compressive forces acting at all points of the spherical surface which tend to crush it:



The magnitude of the compressive force acting on any little element of area of the sphere is essentially constant. This force magnitude divided by the area over which it acts is the hydrostatic pressure, $\mathcal{P}_o$.



To get a feel for the magnitudes of hydrostatic pressures found in ordinary circumstances, consider the following:

(1) Near sea level the hydrostatic pressure of the air is $\mathcal{P}_o \doteq 14.7\ \mathrm{lb/in^2} \equiv 1$ atmosphere of pressure.

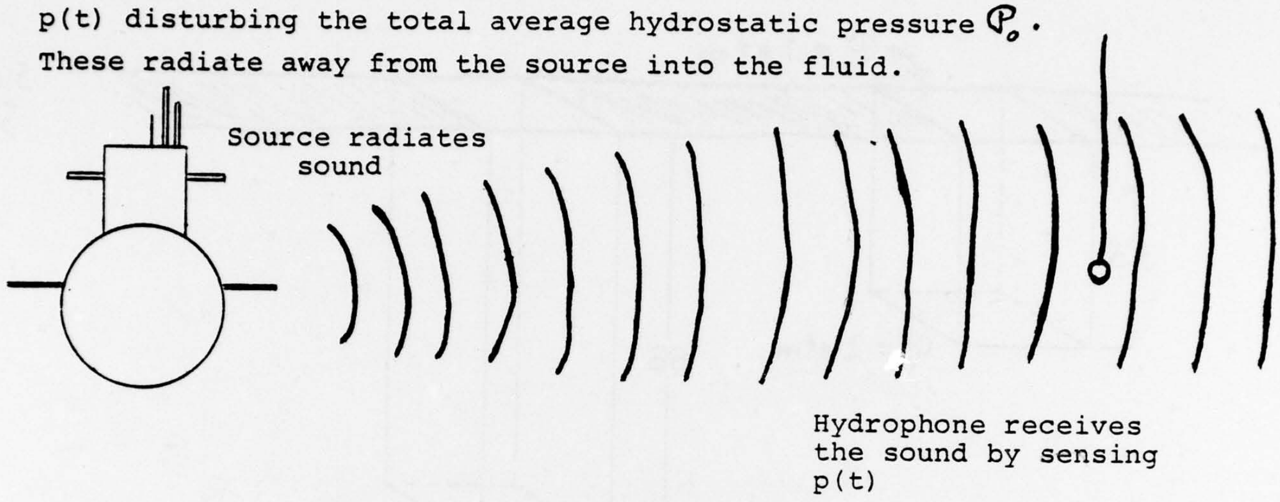
(2) Just beneath the surface of the ocean the pressure is also $\mathcal{P}_o \doteq 14.7\ \mathrm{lb/in^2}$.

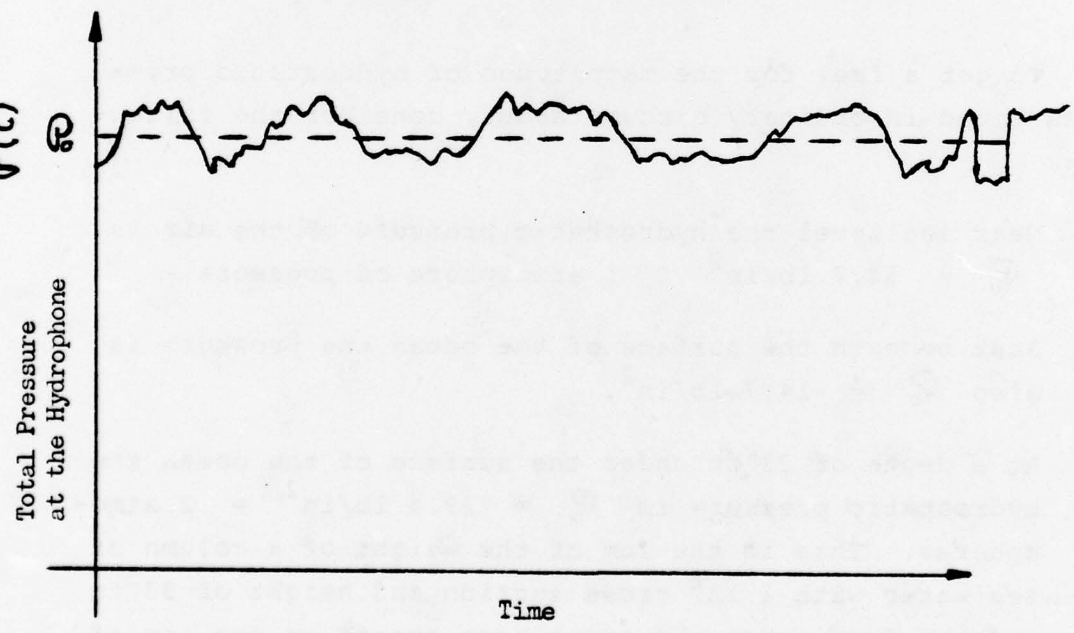
(3) At a depth of 33 ft under the surface of the ocean the hydrostatic pressure is $\mathcal{P}_o = 29.4\ \mathrm{lb/in^2} = 2$ atmospheres. This is the sum of the weight of a column of sea water with 1 in^2 cross section and height of 33 ft and the force that the atmosphere exerts on the top of the water column.

(4) In like manner, at a depth of 66 ft the total hydrostatic pressure is $44.1\ \mathrm{lb/in^2} = 3$ atmospheres.

ACOUSTIC PRESSURE

Sound is composed of very small pressure fluctuations p(t) disturbing the total average hydrostatic pressure $\mathcal{P}_o$. These radiate away from the source into the fluid.





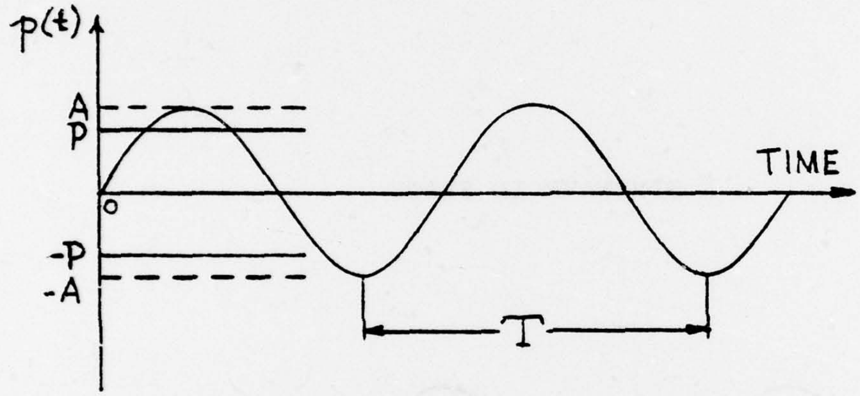
ACOUSTIC PRESSURE $= \mathcal{P}(t) - \mathcal{P}_o = p(t)$

p(t) is the quantity sensed by most hydrophones, since they are sensitive to <u>*changes*</u> *in the total pressure.*

MONOFREQUENCY SIGNALS

For ease of discussion, we will at first restrict ourselves to consideration of <u>tonals</u> (monofrequency signals). Later on, we will see that more complicated signals can be broken down into a collection of monofrequency signals.

The time history of the acoustic pressure at a given point in space for a monofrequency signal can be represented as a sine wave,

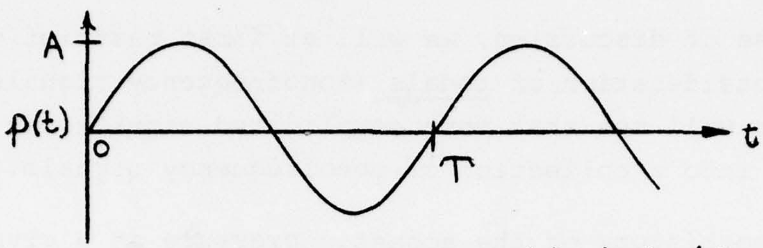


P = The effective amplitude of the acoustic pressure. This is not the same as the <u>peak</u> amplitude A, but is more convenient. It turns out that $P = A/\sqrt{2} = 0.707A$ for monofrequency signals.

T = The period of the monofrequency signal. The period is the time interval required for the acoustical signal to go through one complete cycle, returning to the configuration it had at the beginning of the interval. The frequency f of the wave is the number of cycles per second, and is given by $f = 1/T$.

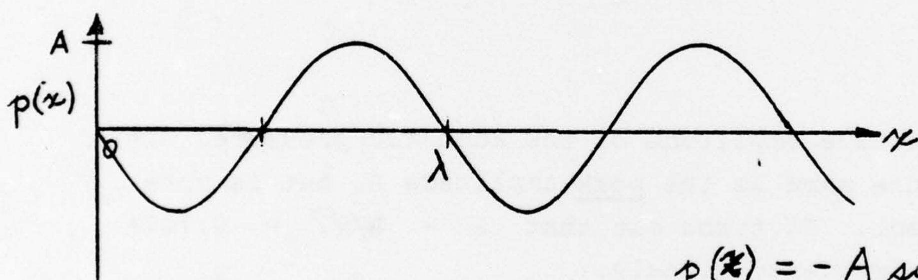
Notice that $\mathcal{P}$ *and* P *represent totally different quantities;* $\mathcal{P}$ *is the instantaneous* <u>*hydrostatic*</u> *pressure and* P *is the effective amplitude of the* <u>*acoustic*</u> *pressure.*

A SINEWAVE IN TIME



$$p(t) = A \sin\left(2\pi \frac{t}{T}\right)$$

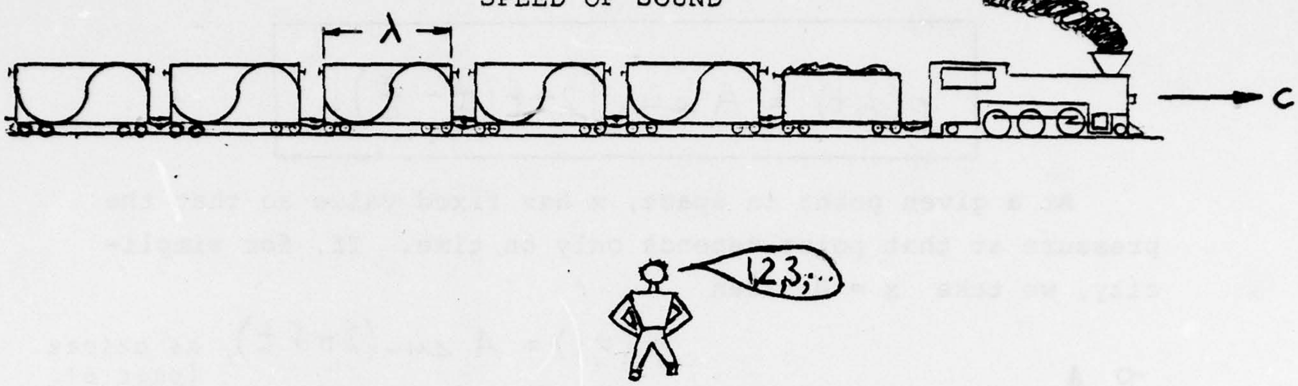
A SINEWAVE IN SPACE



$$p(x) = -A \sin\left(2\pi \frac{x}{\lambda}\right)$$

λ = the wavelength of the signal. It is the distance over which the signal goes through one complete cycle.

SPEED OF SOUND



A wave advancing through space is like a train traveling along the tracks:

The length of each boxcar is the wavelength λ, and the time it takes each boxcar to pass the observer is the period T.

The train advances a distance λ in a time T, so its speed c must be λ/T.

$$c = \lambda/T$$

Since the number of cars passing the observer in unit time (1 sec) is the frequency $f = 1/T$,

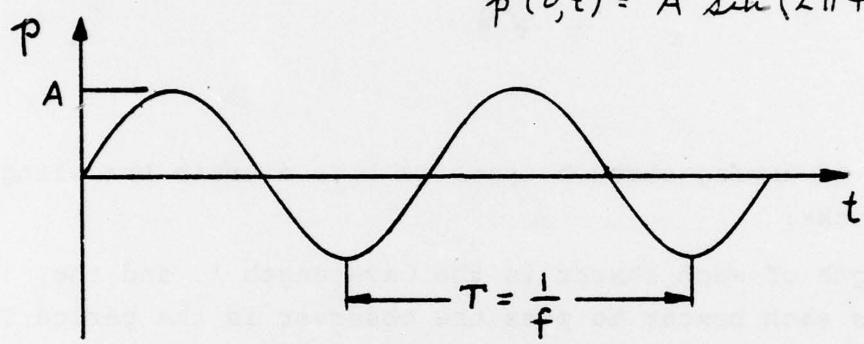
$$\boxed{\lambda f = c}$$

A TRAVELING WAVE

$$\boxed{p(x,t) = A \sin\left[2\pi f\left(t - \frac{x}{c}\right)\right]}$$

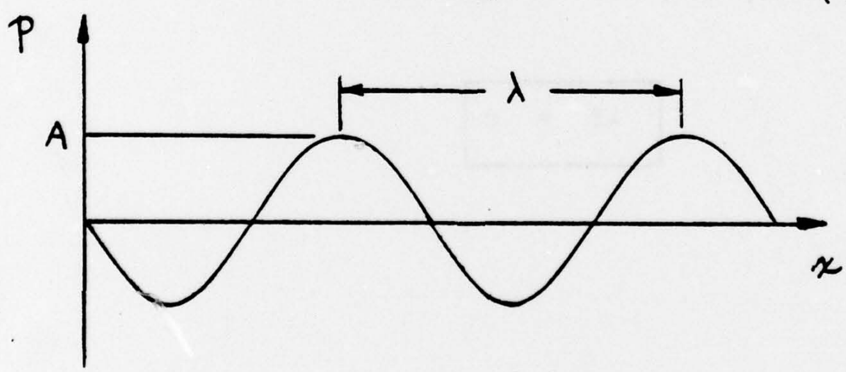
At a given point in space, x has fixed value so that the pressure at that point depends only on time. If, for simplicity, we take $x = 0$, then

$$p(0,t) = A \sin(2\pi f t)$$ as before (page 6).



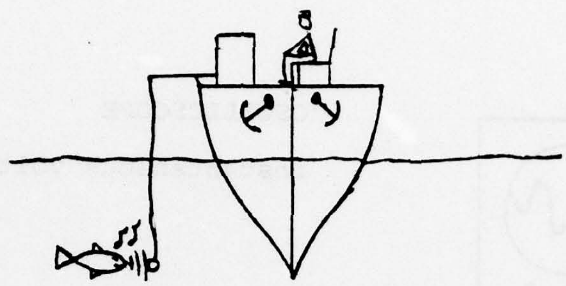
On the other hand, if we look at the wave at one instant (i.e., take a picture of it) then t = constant, and p depends only on x. If, for simplicity, we take $t = 0$, then

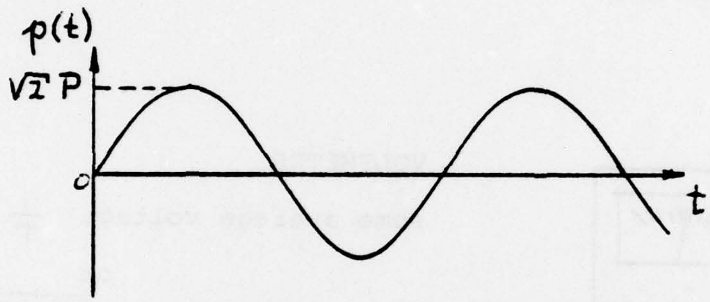
$$p(x,0) = -A \sin\left(\frac{2\pi}{\lambda} x\right)$$ again as before (page 6).



If we keep track of a given part of the wave, then as time increases this part moves towards larger x with speed c.

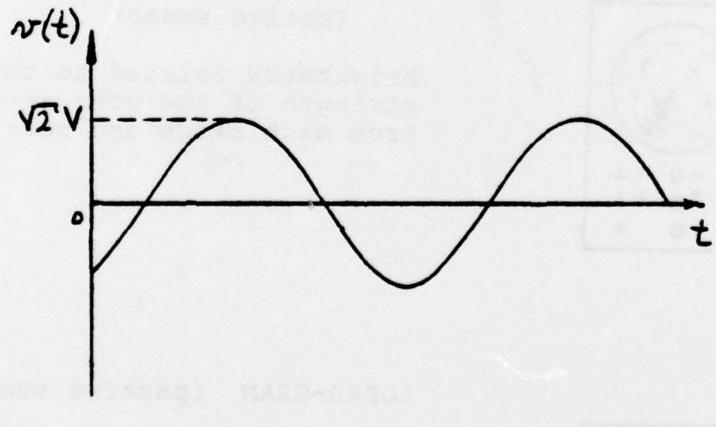
HYDROPHONE OUTPUT VOLTAGE



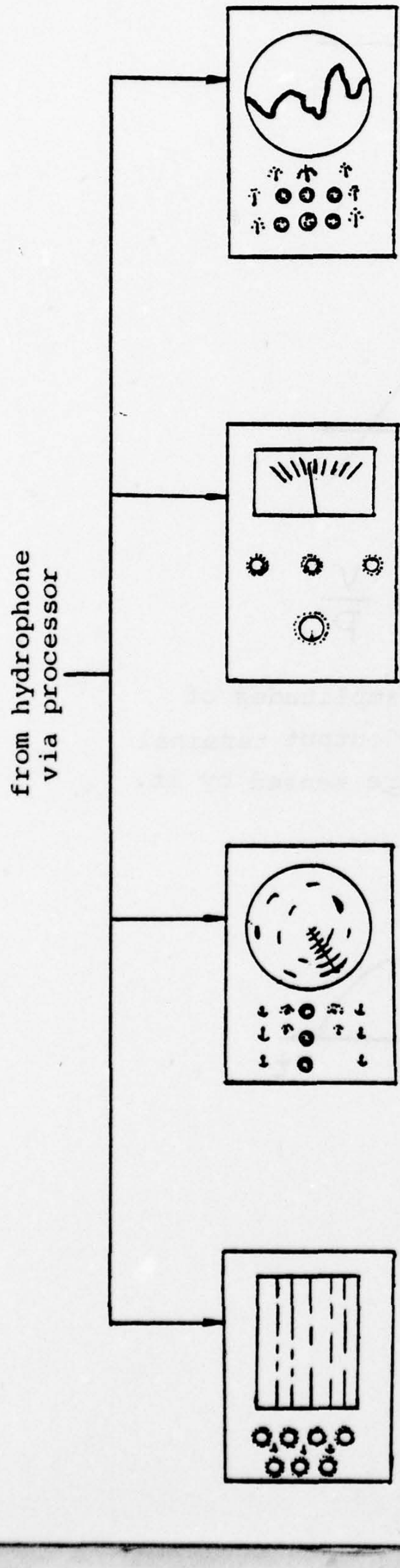


Hydrophone Sensitivity $\mathcal{M} = \frac{V}{P}$

where V and P are the effective amplitudes of the instantaneous voltage at the output terminal of the hydrophone and the pressure sensed by it.



DISPLAYS



OSCILLOSCOPE

instantaneous voltage $v(t)$

VOLTMETER

some average voltage $\frac{1}{T}\int_0^T |v|\,dt$

or

effective voltage $\sqrt{\frac{1}{T}\int_0^T v^2\,dt}$

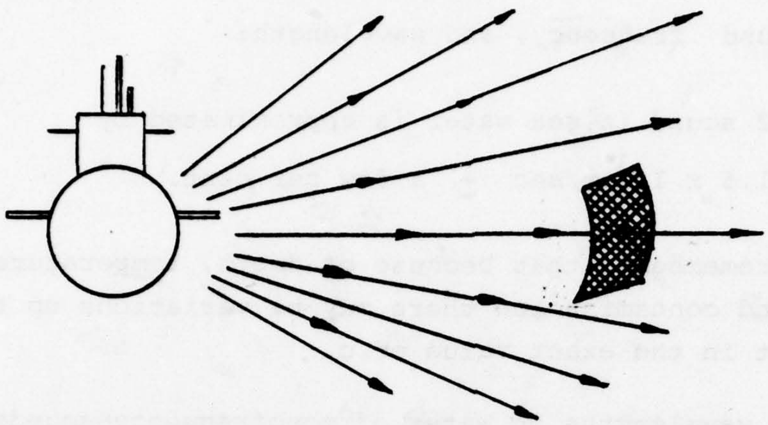
BEARING RANGE RECORDER
(active sonar)

brightness related to the strength of the echo returned from each range and bearing

LOFAR-GRAM (passive sonar)

paper is electrically burnt, with darkness related to the strength of the signal received at each frequency and time

INTENSITY



Intensity = energy crossing unit area in unit time

where "area" is the area of a "window" perpendicular to the direction of travel of the wave.

In terms of measurable quantities,

$$\boxed{\text{Intensity } I = \frac{P^2}{\rho c}}$$

where ρ = density = mass per unit volume of the medium,
and ρc = specific acoustic impedance of the fluid.

	Density	Speed of Sound	ρc
Sea Water	1026 kg/m^3	1500 m/sec	1.54×10^6 Rayls
Air	1.21 kg/m^3	343 m/sec	415 Rayls

1 Rayl = 1 $kg/(m^2\text{-sec})$

See Appendix A if scientific notation ($N \times 10^M$) is new to you.

SIZES OF THINGS IN WATER

Speed of sound, frequency, and wavelength:

The speed of sound in sea water is approximated by

$$c \sim 1.5 \times 10^3 \text{ m/sec} \pm \text{ a few per cent.}$$

It must be remembered that because of depth, temperature, salinity, and contamination there may be variations up to a few per cent in the exact value of c.

Approximate wavelengths in water of monofrequency sounds of different frequencies are:

Frequency	Wavelength
10 Hz = 10 cps	150 m
50	30
100	15
500	3
1 kHz = 1000 cps	1.5
5	0.3
10	0.15
50	0.03 = 3 cm

1 m = 1.09 yd

Pressure amplitudes:

$$1 \text{ atm} \doteq 10^6 \ \mu b$$

$$1 \ \mu b = 10^5 \ \mu Pa$$

$\mathcal{P}_o$ = 1 atmosphere = 14.7 lb/in^2 at the ocean surface

$\mathcal{P}_o$ increases by about 1 atm for every 33 ft of depth in water.

Representative acoustic pressures generated near a sonar transmitter:

$$P \sim 10^4 \text{ to } 10^6 \ \mu b \text{ or } 0.01 \text{ to } 1 \text{ atm.}$$

If $P \sim \mathcal{P}_o/\sqrt{2}$ then <u>cavitation</u> is likely and performance may be degraded. Representative weak acoustic pressures detectable by a sonar receiver:

$$P \sim 10^{-2} \ \mu b \text{ or } 10^{-8} \text{ atm.}$$

CAVITATION

Ocean waters contain dissolved gases.

If a sample of sea water is depressurized, it is observed that the dissolved gases can come out of solution, forming bubbles. If the pressure is restored, these bubbles will collapse.

This process can occur when a high amplitude acoustical wave passes through water. If the peak acoustic pressure amplitude is greater than a certain value, then the negative acoustic pressure causes the water to "boil" and give up its dissolved gases in the formation of bubbles. When the same portion of water is then compressed by the positive portion of the wave, the bubbles collapse suddenly. The sound of these collapses resembles that of frying fat.

Cavitation corrodes surfaces on which it occurs, and transforms acoustical energy to heat. Cavitation sets a natural limit on the useful strength of sound waves in water.

For frequencies below about 10 kHz, and pulses longer than 100 millsec, the peak acoustic pressure amplitude for cavitation is about the same value as the total hydrostatic pressure $\mathcal{P}_o$.

SOUND PRESSURE LEVEL = SPL

$$\boxed{\text{SPL re } P_{ref} \;=\; 20 \,\ell og\, (P/P_{ref})}$$

$$\boxed{P_{ref} \;=\; \begin{array}{ll} 0.0002\ \mu b & \text{(old noise measurements)} \\ 1 \mu b & \text{(old conventional standard)} \\ 1 \mu Pa & \text{(new Navy standard)} \end{array}}$$

Both P and P_{ref} are effective pressure amplitudes.

The unit of SPL is the decibel (dB).

<u>Examples</u>

$$\text{If } P \;=\; 10^6 \mu b \;\doteq\; 1 \text{ atm, then SPL} = \begin{cases} 194 \text{ dB re } 0.0002 \mu b \\ 120 \text{ dB re } 1 \mu b \\ 220 \text{ dB re } 1 \mu Pa \end{cases}$$

$$\text{If } P \;=\; 10^2 \mu b \;\doteq\; 10^{-4} \text{ atm, then SPL} = \begin{cases} 114 \text{ dB re } 0.0002 \mu b \\ 40 \text{ dB re } 1 \mu b \\ 140 \text{ dB re } 1 \mu Pa \end{cases}$$

$$\text{If } P \;=\; 10^{-2} \mu b \;\doteq\; 10^{-8} \text{ atm, then SPL} = \begin{cases} 34 \text{ dB re } 0.0002 \mu b \\ -40 \text{ dB re } 1 \mu b \\ 60 \text{ dB re } 1 \mu Pa \end{cases}$$

If the reader is not used to working with log to the base 10, he should consult Appendix A.

CONVERSION BETWEEN SPL's WITH DIFFERENT REFERENCE PRESSURES

For a given pressure amplitude

$$\text{SPL re } 1\mu\text{Pa} = \text{SPL re } 1\mu\text{b} + 100 \text{ dB}$$

$$\text{SPL re } 0.0002\mu\text{b} = \text{SPL re } 1\mu\text{b} + 74 \text{ dB}$$

Notice

0 dB does not mean the absence of sound, but means $P = P_{ref}$.

If $P = P_{ref}$,

then

$$\text{SPL} = 20\,\ell og(P/P_{ref}) = 20\,\ell og(P_{ref}/P_{ref}) = 20\,\ell og\,1 = 0 \text{ dB re } P_{ref}$$

INTENSITY LEVEL = IL

$$\boxed{\text{IL re } I_{ref} = 10 \, \ell og(I/I_{ref})}$$

If $\quad I_{ref} = P_{ref}^{\;2}/(\rho c)$ and $\quad I = P^2/(\rho c)$

then $\quad 10 \, \ell og \; I/I_{ref} = 10 \, \ell og \; P^2/P_{ref}^{\;2} = 20 \, \ell og \; P/P_{ref}$

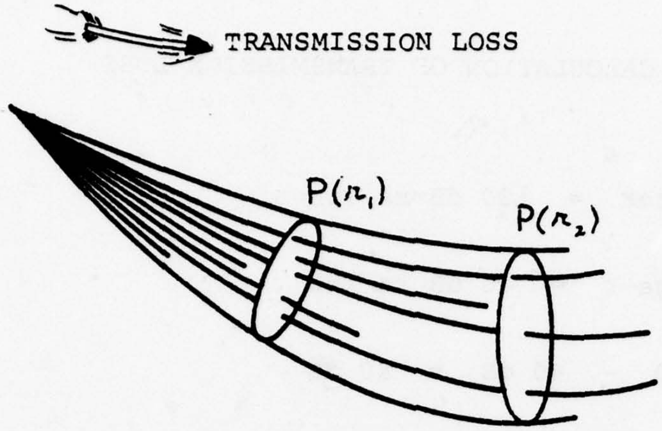
Thus

$$\boxed{\text{IL re } I_{ref} = \text{SPL re } P_{ref} \quad \text{if} \quad I_{ref} = P_{ref}^{\;2}/(\rho c)}$$

Notice that the <u>level</u> of a signal in dB is obtained by 20 ℓog (Pressure Ratio) or 10 ℓog (Intensity Ratio)

For our purposes, IL and SPL are totally equivalent when I_{ref} and P_{ref} are related by

$$I_{ref} = P_{ref}^{\;2}/(\rho c)$$



$$SPL(r_1) = 20 \log \frac{P(r_1)}{P_{ref}}$$

$$SPL(r_2) = 20 \log \frac{P(r_2)}{P_{ref}}$$

$$SPL(r_1) - SPL(r_2) = 20 \log \frac{P(r_1)}{P(r_2)}$$

$$\boxed{TL = SPL\ (1\ \text{meter}) - SPL\ (r\ \text{meters}) = 20 \log \frac{P(1)}{P(r)}}$$

The Transmission Loss TL is a measure of the reduction in SPL of a signal as it traverses the distance between a point 1 meter to a point r meters from the source. Transmission Loss is referred to the SPL at 1 meter: TL = 0 dB at r = 1 m. [For large sources with complicated "near fields" the SPL (1 meter) must be found by extrapolating back from the less complicated region at distances larger than 1 meter.]

SAMPLE CALCULATION OF TRANSMISSION LOSS

If SPL at 1 meter = 120 dB re 1μb

and SPL at range r = 40 dB re 1μb

then TL = 120 - 40 dB = 80 dB .

TL *does <u>not</u> depend on the reference pressure.*

Let's convert the above SPL's to reference 1μPa and recalculate the TL.

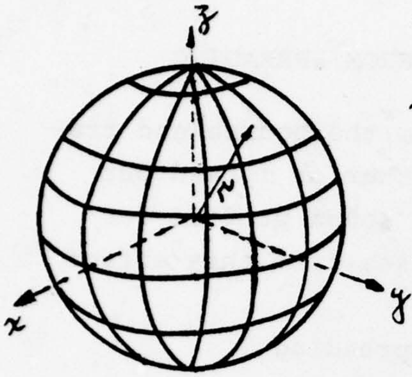
Now SPL at 1 meter = 220 dB re 1μPa

and SPL at range r = 140 dB re 1μPa

so that TL = 220 - 140 = 80 dB .

SPHERICAL AND CYLINDRICAL WAVES

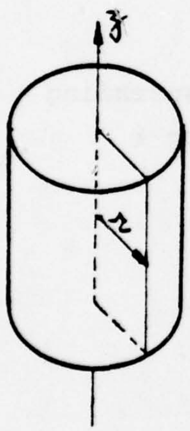
SPHERICAL



$r =$ distance from the center of the sphere

$$p = \frac{A'}{r} \sin\left[2\pi f\left(t - \frac{r}{c}\right)\right].$$

CYLINDRICAL



$r =$ distance from the axis of the cylinder

$$p = \frac{B}{\sqrt{r}} \sin\left[2\pi f\left(t - \frac{r}{c}\right)\right] \quad \text{if } r \gg \lambda.$$

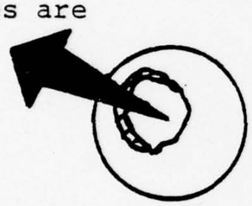
The total transmission loss TL *can be considered to arise from two different causes:*

I. TRANSMISSION LOSS FROM SPREADING

As the rays of sound propagate out from the source and travel through the water, they will bunch together or spread out depending on the properties of the speed of sound profile for the water. The transmission loss which arises from this effect is termed

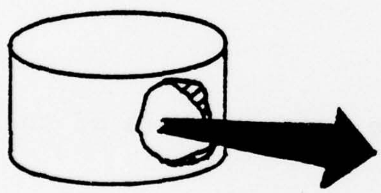
$$TL_g \;=\; \text{transmission loss from spreading}.$$

Some simple examples of spreading for rays traveling straight lines are



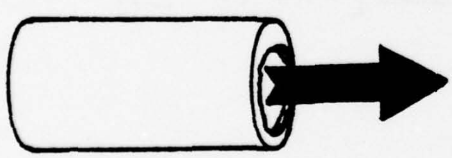
Spherical spreading

$$TL_g = 20 \,\ell og\, r$$



Cylindrical spreading

$$TL_g = 10 \,\ell og\, r$$



No spreading

$$TL_g = 0 \text{ dB}.$$

In the real ocean, the rays never travel in straight lines, so that these above simple examples are unrealistic. We will see, however, that there are many situations in which the sound will spread out spherically or cylindrically over large portions of its path, and the above simple equations will appear in the complete transmission loss equation.

II. TRANSMISSION LOSS FROM DISSIPATION

There are a variety of other physical effects termed dissipative effects, which reduce the SPL of the sounds in addition to the spreading of the rays. Since the spreading of rays is described by TL_g, we will call the portion of the transmission loss which results from these other effects

$$TL_{\ell} \;=\; \text{Transmission Loss from dissipation.}$$

There are a number of these which are important in underwater sound propagation:

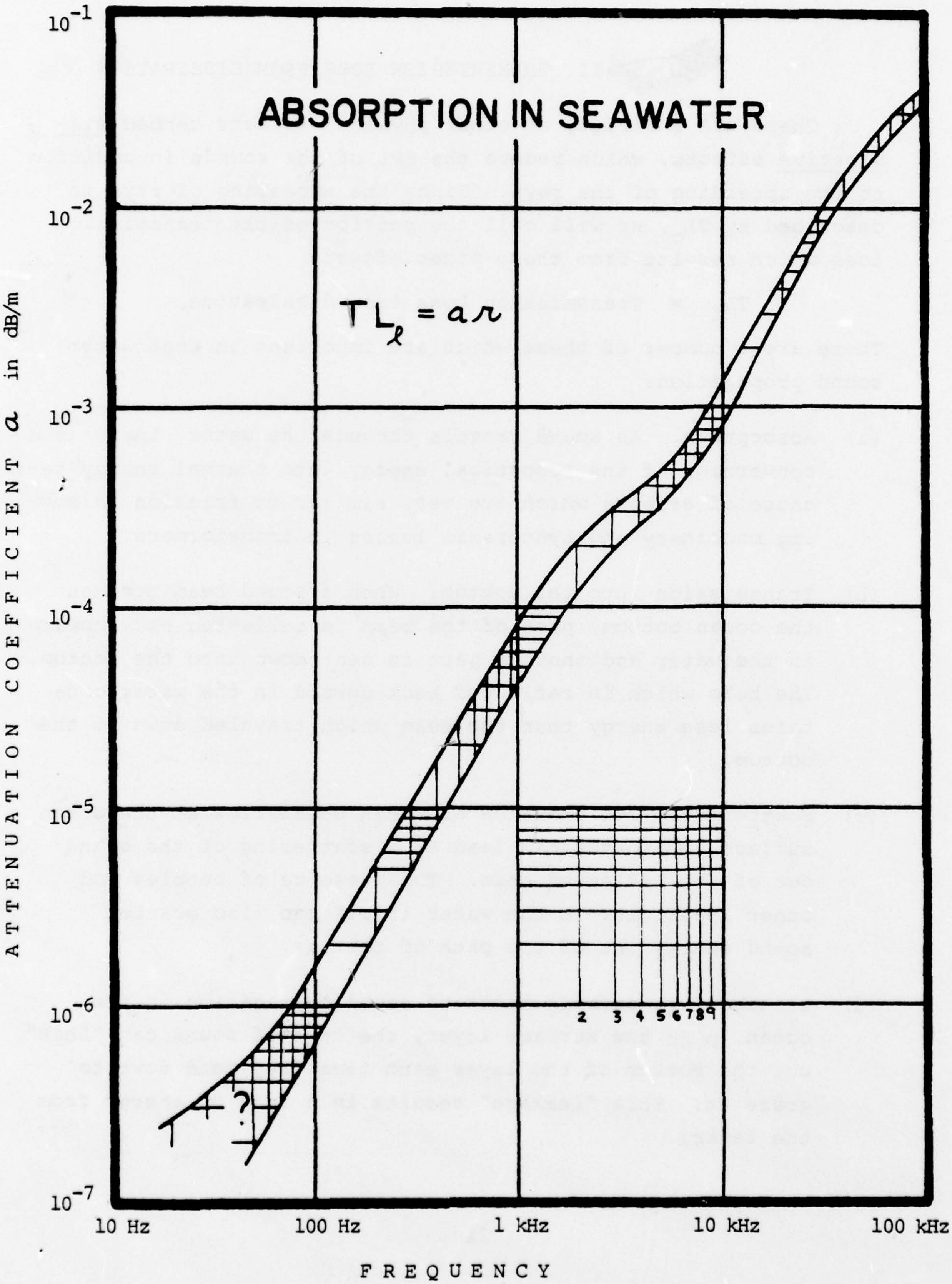
(a) Absorption. As sound travels through the water, there is a conversion of the acoustical energy into thermal energy because of effects which are very similar to friction in moving machinery and hysteresis losses in transformers.

(b) Transmission into the bottom: When a sound beam strikes the ocean bottom, part of the beam is reflected back upward in the water and another part is sent down into the bottom. The beam which is reflected back upward in the water contains less energy than the beam which traveled down to the bottom.

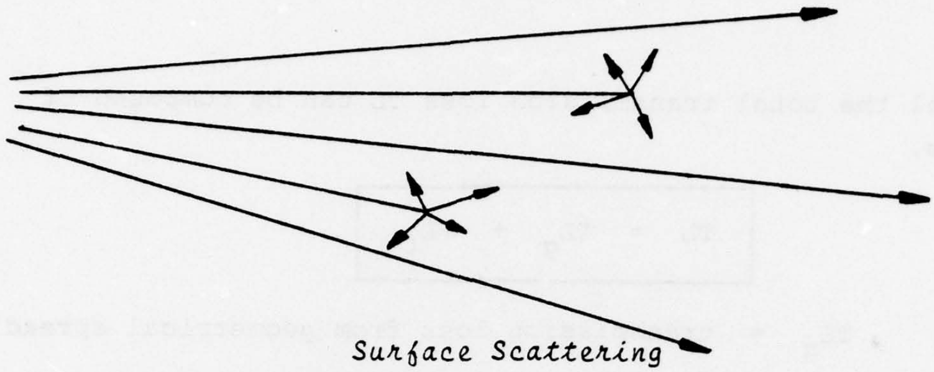
(c) Scattering. The presence of rough boundaries at the ocean surface and bottom can lead to a scattering of the sound out of the reflected beam. The presence of bubbles and other impurities in the water itself can also scatter sound energy out of the path of the ray.

(d) Leakage. In certain kinds of sound propagation in the ocean, as in the surface layer, the rays of sound can "leak" out the bottom of the layer each time they bend down to graze it. This "leakage" results in a loss of energy from the layer.

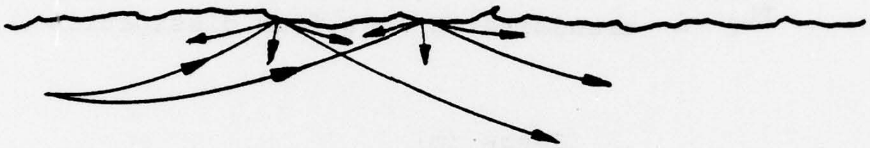
DISSIPATIVE LOSS MECHANISMS



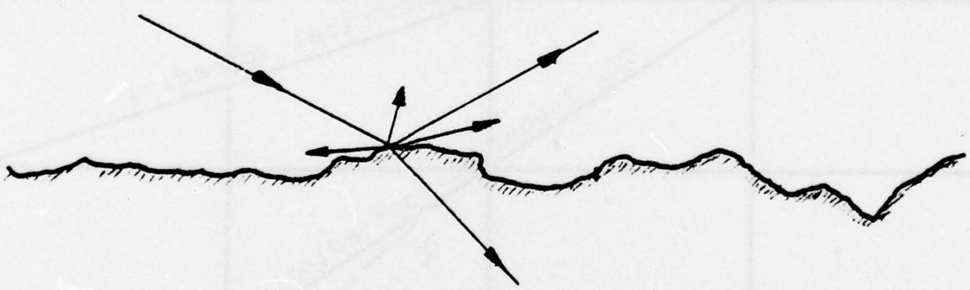
Volume Scattering



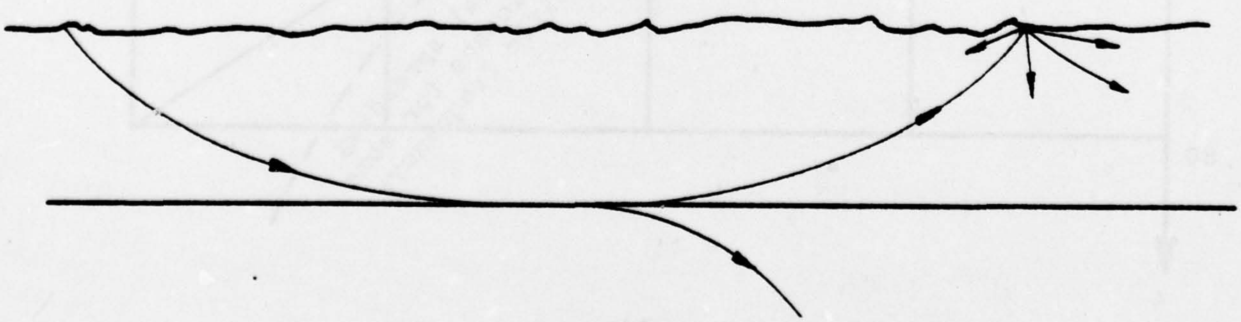
Surface Scattering



Transmission into Bottom with Scattering



Leaky Duct with Scattering



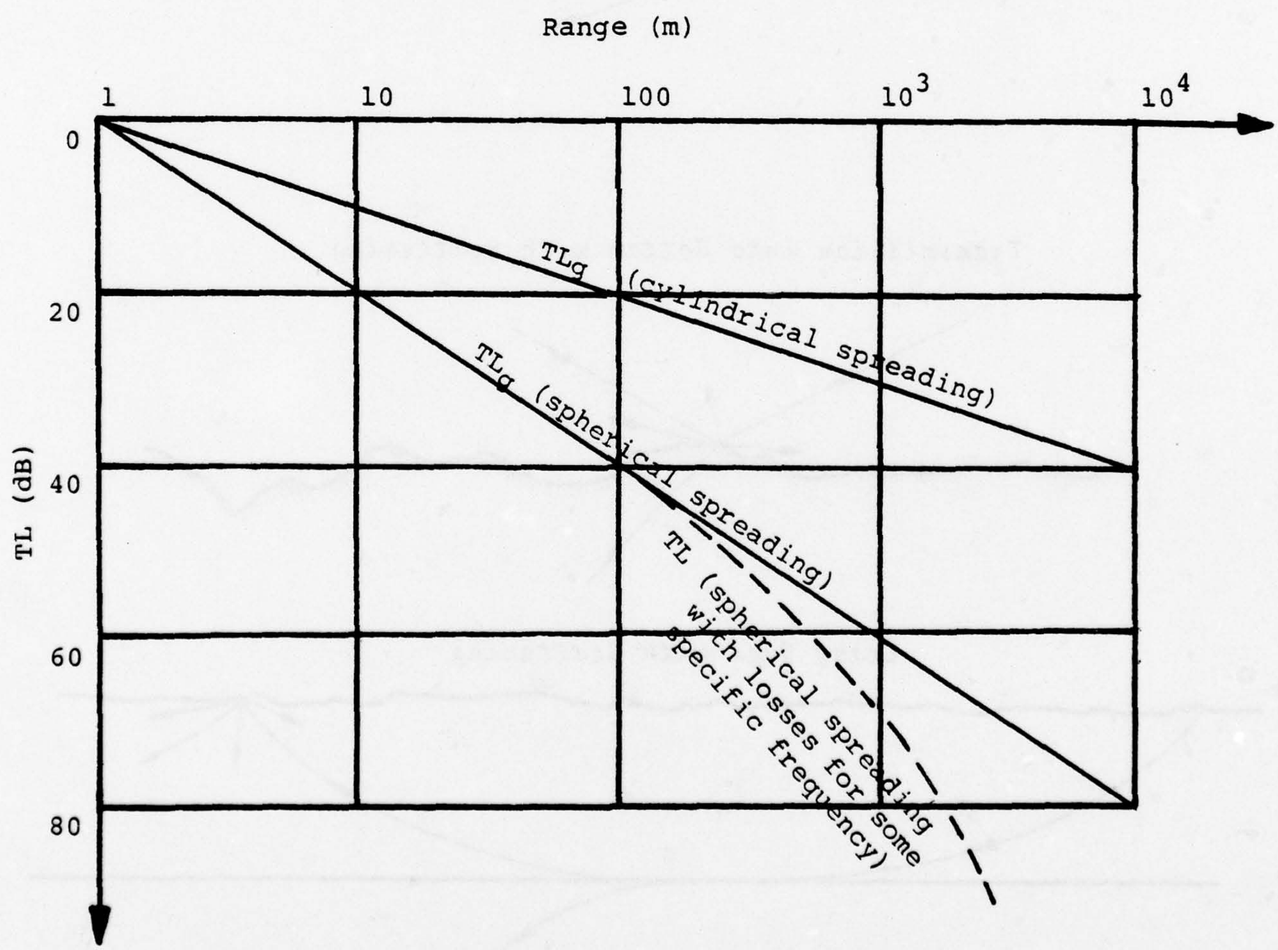
TRANSMISSION LOSS

In general the total transmission loss TL can be composed of two terms,

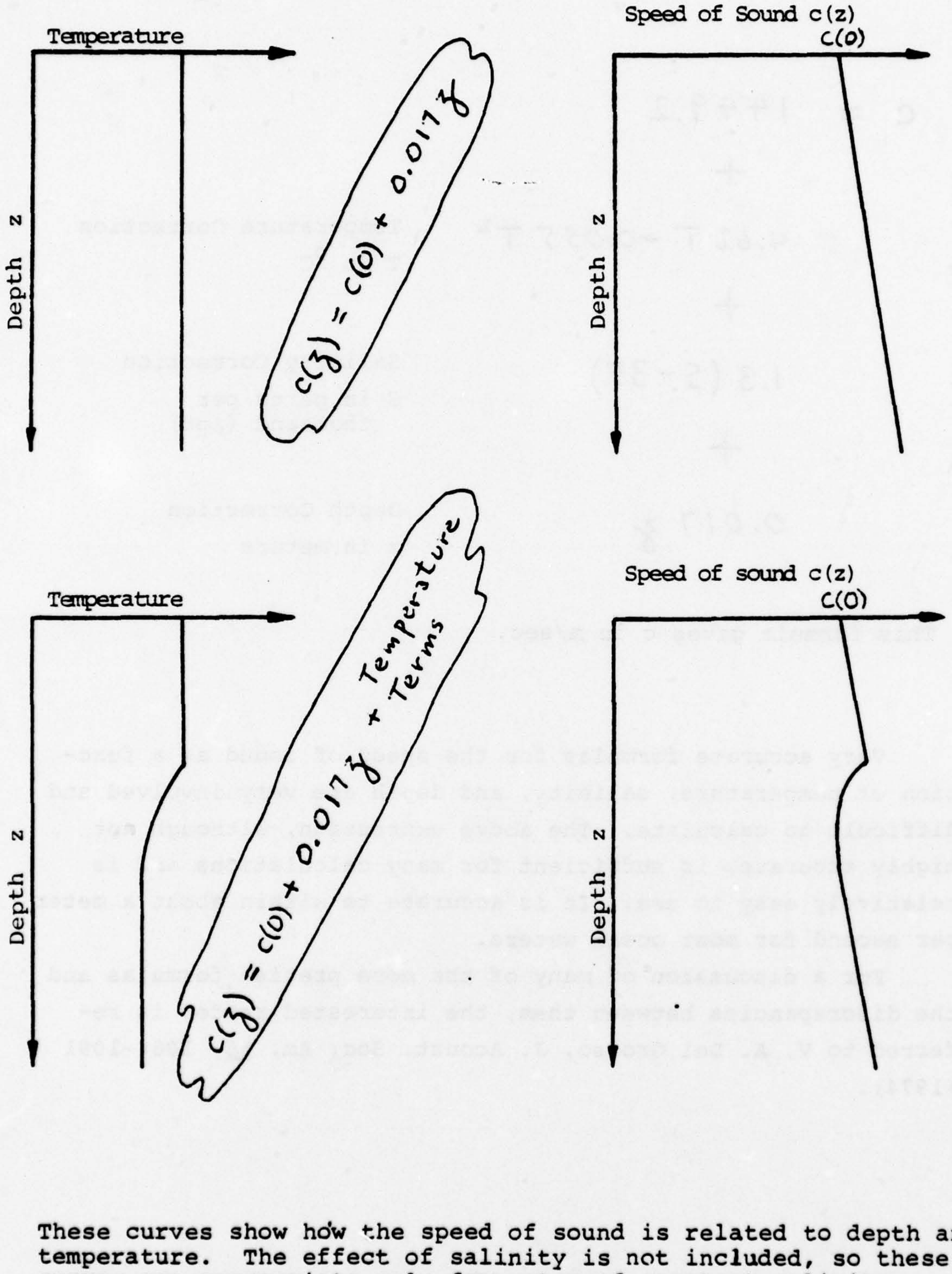
$$\boxed{TL = TL_g + TL_\ell}$$

where TL_g = transmission loss from geometrical spreading

TL_ℓ = transmission loss from dissipation



SPEED OF SOUND, TEMPERATURE, AND DEPTH



These curves show how the speed of sound is related to depth and temperature. The effect of salinity is not included, so these curves are appropriate only for water of constant salinity.

SPEED OF SOUND FORMULA

$$c \approx 1449.2 + 4.62T - 0.055T^2 + 1.3(S-35) + 0.017z$$

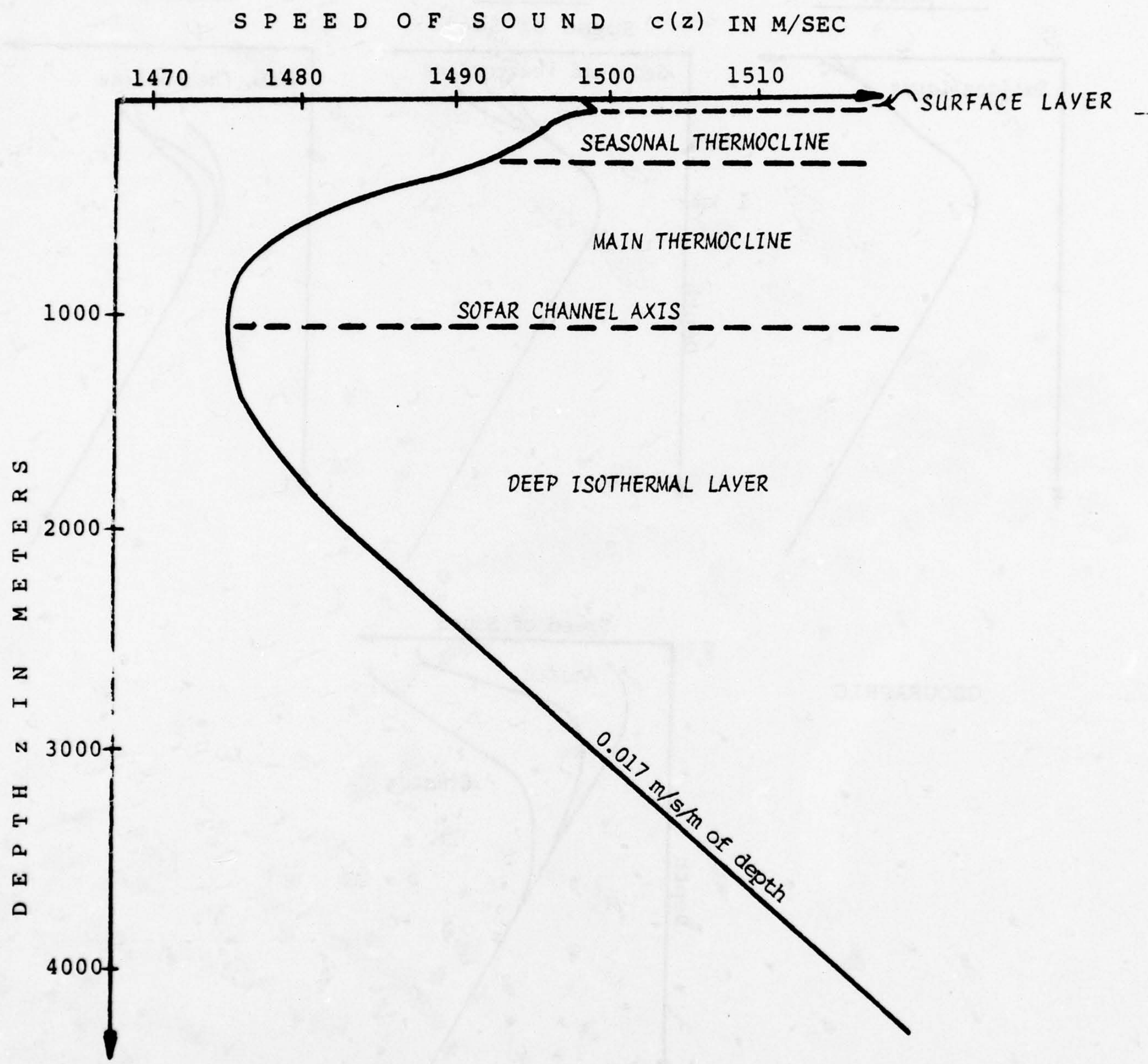
1449.2	
$+\ 4.62T - 0.055T^2$	Temperature Correction T in ^{o}C
$+\ 1.3(S-35)$	Salinity Correction S in parts per thousand (ppt)
$+\ 0.017z$	Depth Correction z in meters

This formula gives c in m/sec.

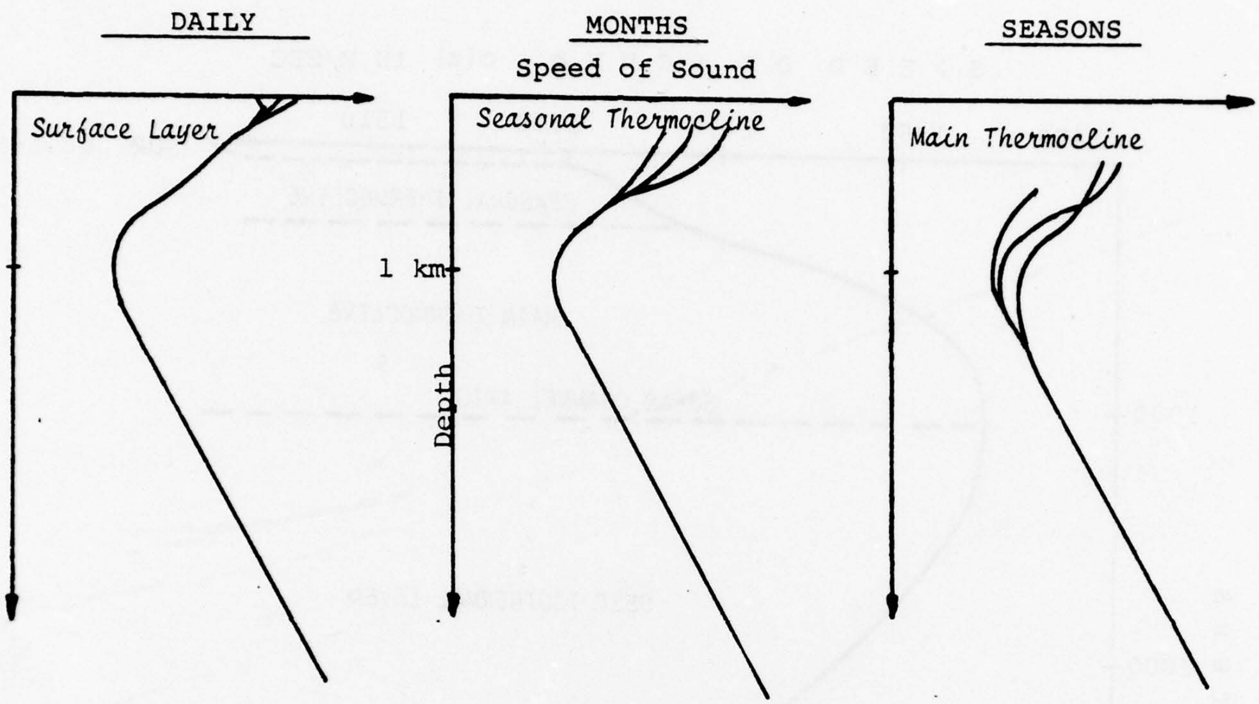
Very accurate formulas for the speed of sound as a function of temperature, salinity, and depth are very involved and difficult to calculate. The above expression, although not highly accurate, is sufficient for many calculations and is relatively easy to use. It is accurate to within about a meter per second for most ocean waters.

For a discussion of many of the more precise formulas and the discrepancies between them, the interested reader is referred to V. A. Del Grosso, J. Acoust. Soc. Am. 56, 1084-1091 (1974).

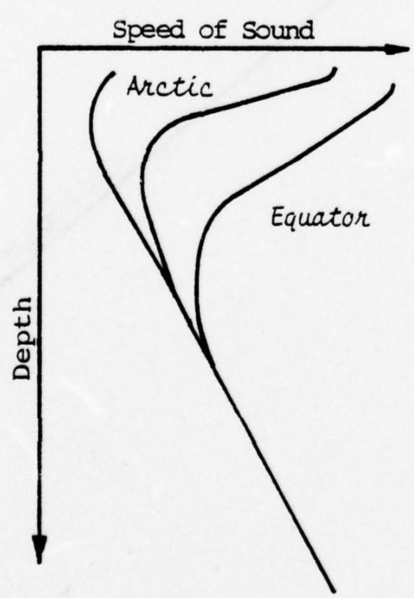
TYPICAL SOUND SPEED PROFILE



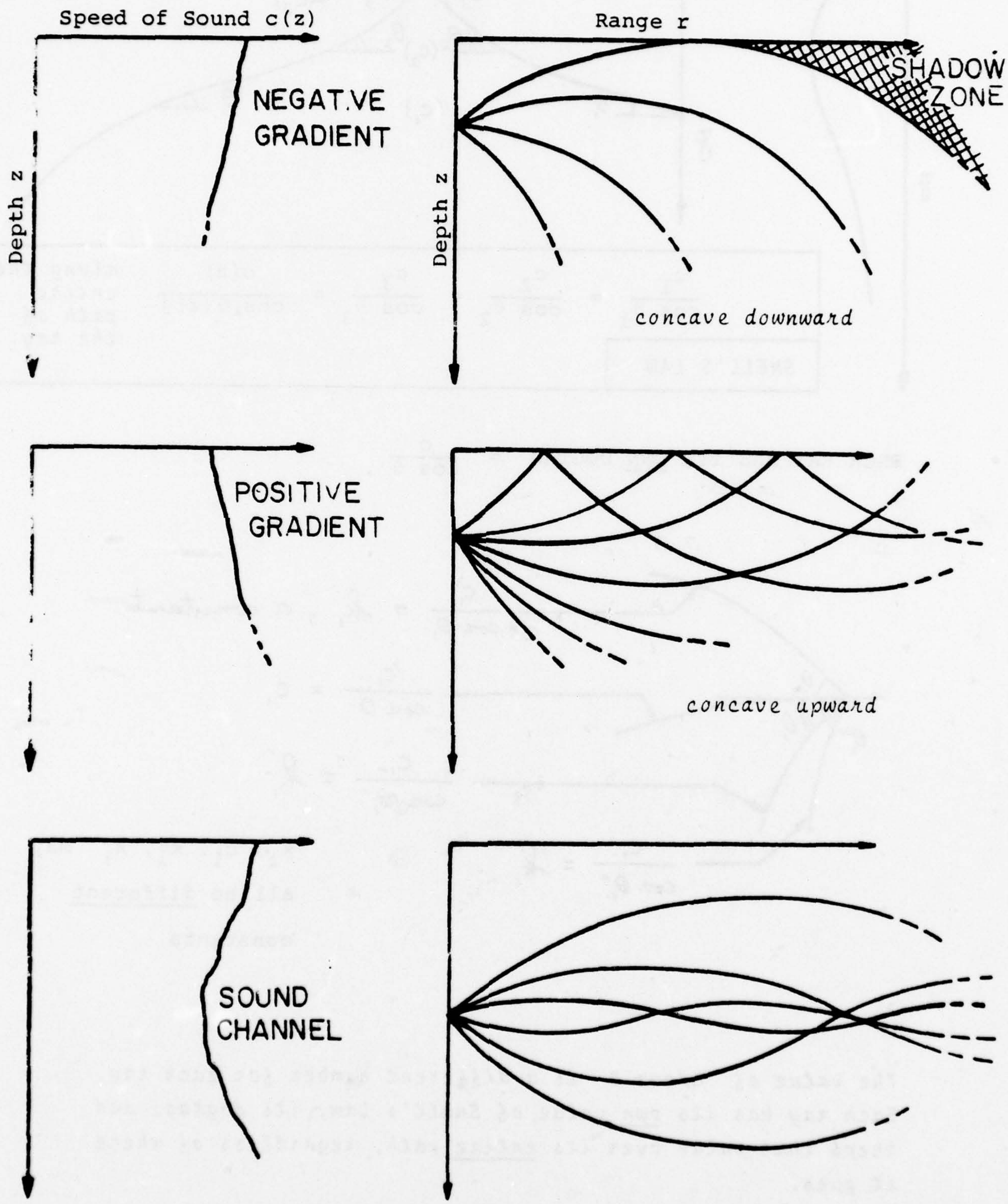
SPEED OF SOUND
PROFILE VARIATIONS



GEOGRAPHIC

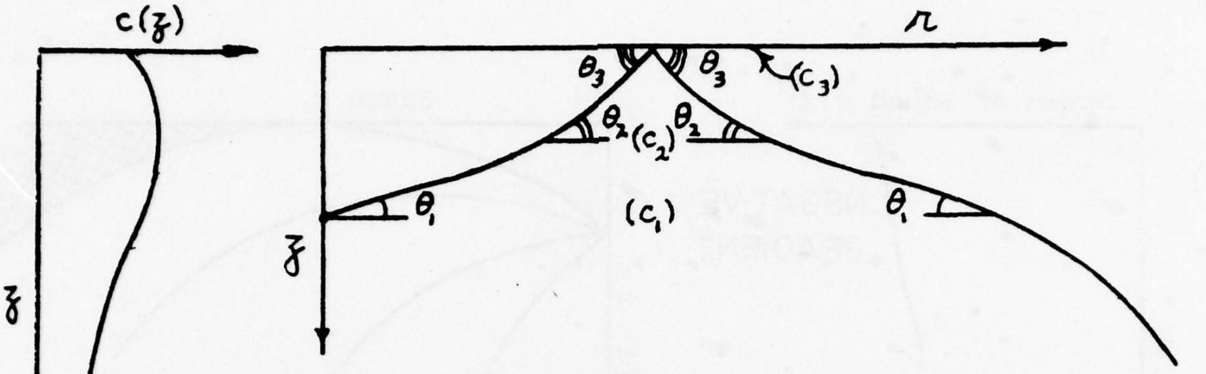


RAYS



Speed of Sound c(z)
Range r
NEGAT.VE GRADIENT
SHADOW ZONE
Depth z
Depth z
concave downward
POSITIVE GRADIENT
concave upward
SOUND CHANNEL

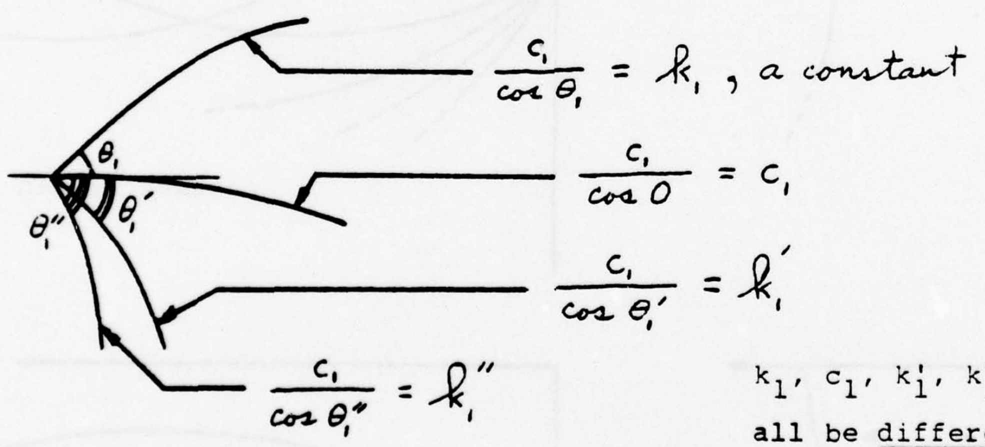
SNELL'S LAW



$$\frac{c_1}{\cos\theta_1} = \frac{c_2}{\cos\theta_2} = \frac{c_3}{\cos\theta_3} = \frac{c(z)}{\cos[\theta(z)]}$$ *along the entire path of the ray.*

SNELL'S LAW

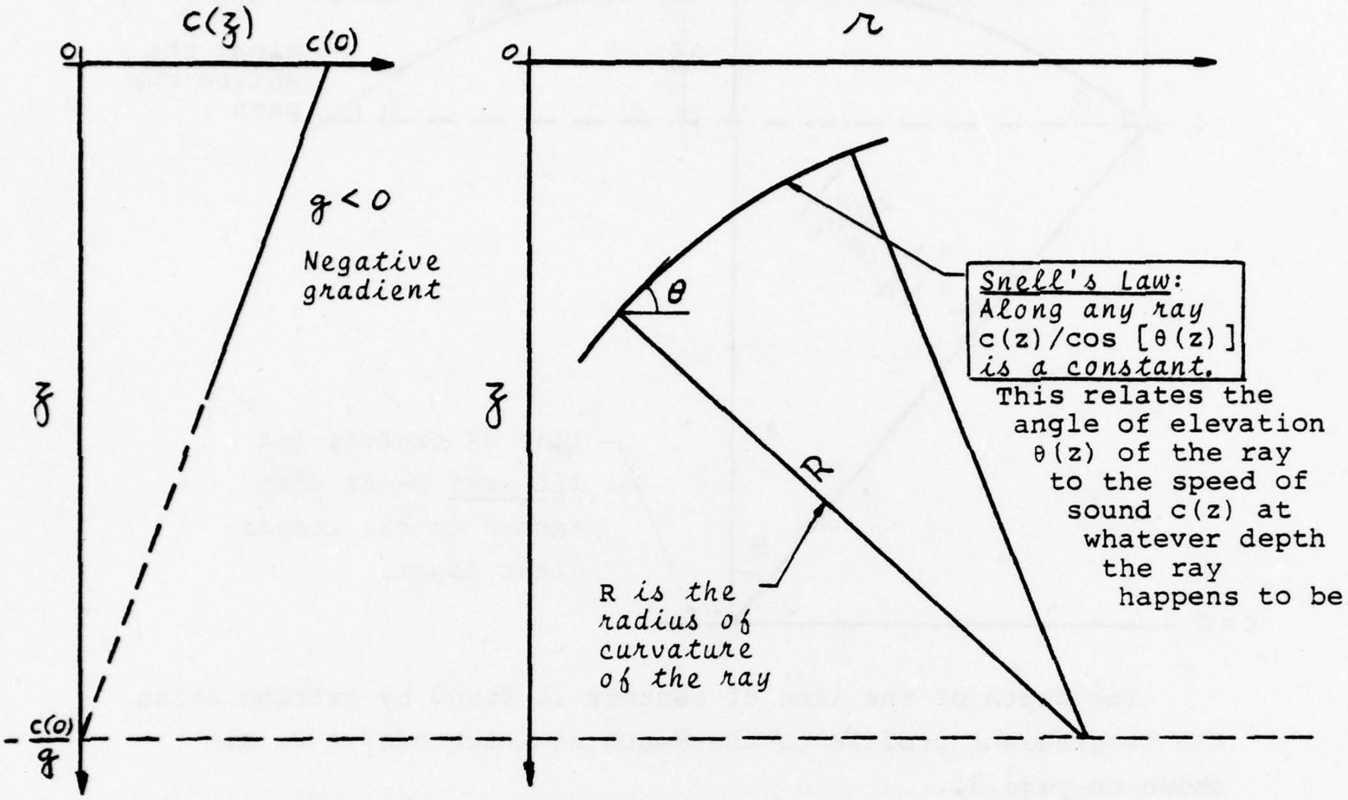
EACH RAY HAS ITS <u>OWN</u> DOGTAG $= \dfrac{c}{\cos\theta}$.



k_1, c_1, k_1', k_1'' may all be <u>different</u> constants

The value of c/cos θ *is a different number for each ray. Each ray has its <u>own</u> value of Snell's Law, its dogtag, and keeps that value over its <u>entire</u> path, regardless of where it goes.*

RAYS IN A CONSTANT GRADIENT (ISOGRADIENT) LAYER



$c(z) = c(0) + gz$	g *is the* *gradient*

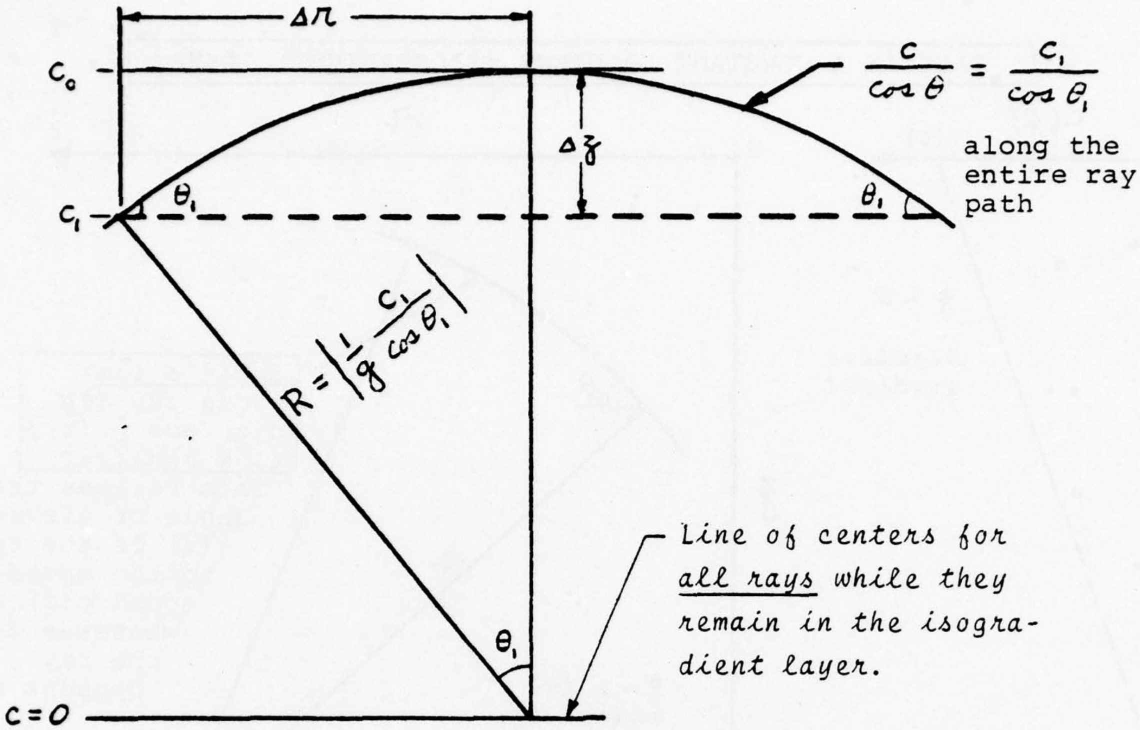
The gradient g can be either positive or negative. For a speed of sound profile composed of layers within each of which the gradient g has constant value, then the segment of the ray within each layer is the arc of a circle whose center lies at a depth $z = -c(o)/g$ below the depth where the speed of sound is c(o). The radius of the circle is

$$R = \left| \frac{1}{g} \frac{c}{\cos\theta} \right|$$

The ray always bends toward the region of lower speed of sound.

The gradient is $$g = \frac{c(z_2) - c(z_1)}{z_2 - z_1} = \frac{\Delta c}{\Delta z}$$

Where z_1 and z_2 are any two depths in the isogradient layer and $c(z_1)$ and $c(z_2)$ the associated speeds of sound at these depths.



The depth of the line of centers is found by extrapolating the isogradient profile to the depth at which c(z) = 0, as shown on page 31.

The gradient is $$g = \frac{c_1 - c_o}{\Delta z}.$$

Snell's Law gives (*At* c_o, $\theta_o = 0$ *and* $\cos\theta_o = 1$.)

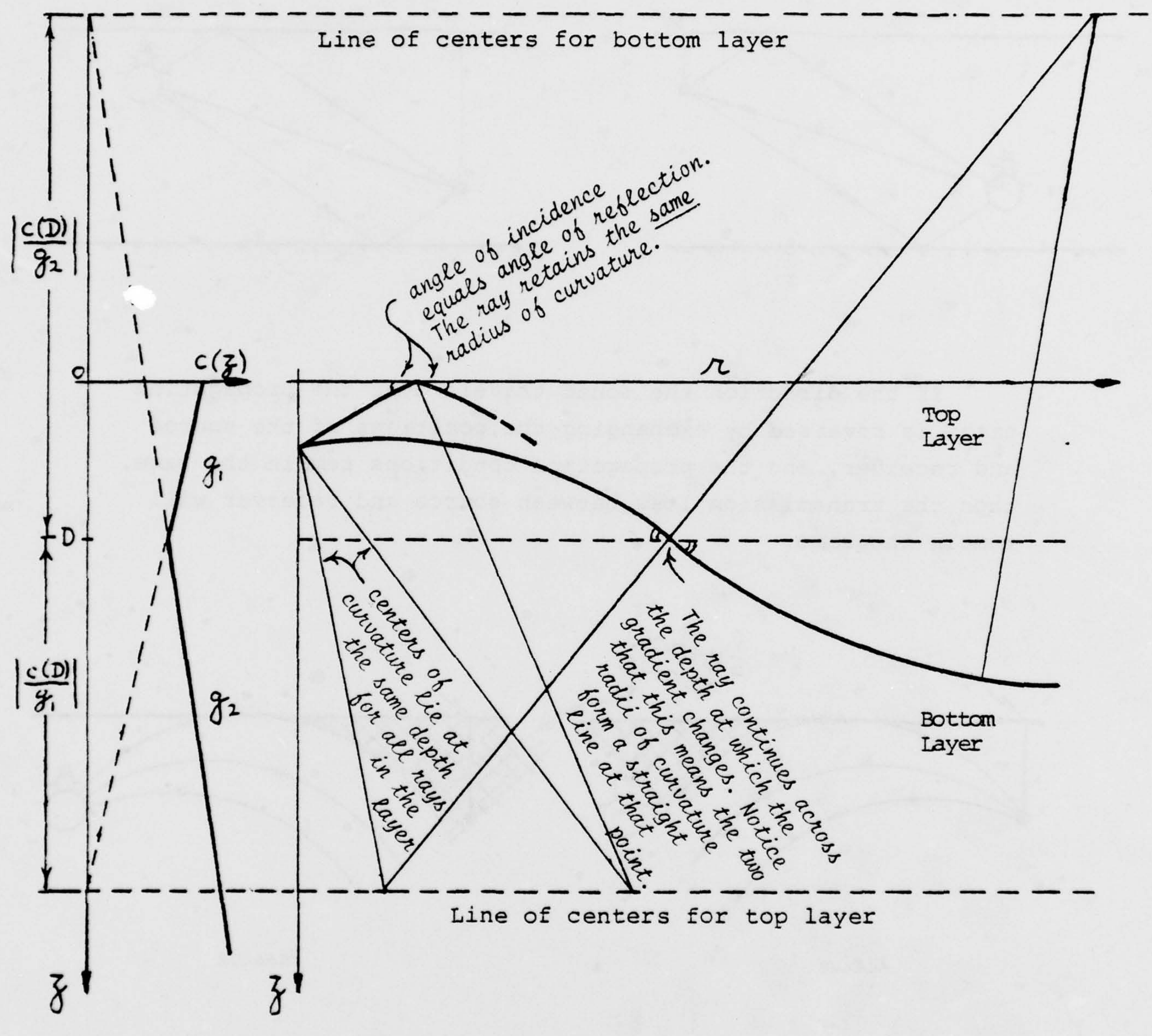
$$\frac{c_1}{\cos\theta_1} = c_o$$

The radius of curvature is $$R = \left|\frac{1}{g}\frac{c_1}{\cos\theta_1}\right| = \left|\frac{c_o}{g}\right|$$

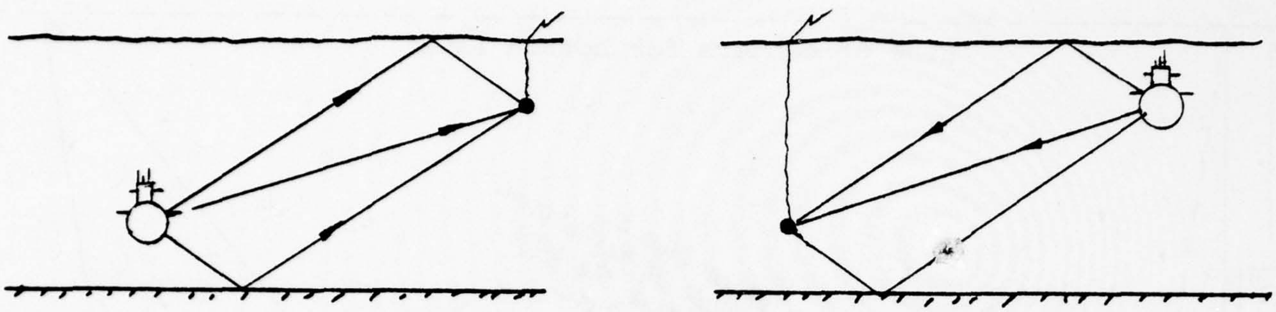
NOTICE THAT TRIGONOMETRY GIVES

$$\boxed{\Delta r = \left|R \sin\theta_1\right|} \qquad \boxed{\Delta z = R(1-\cos\theta_1)}$$

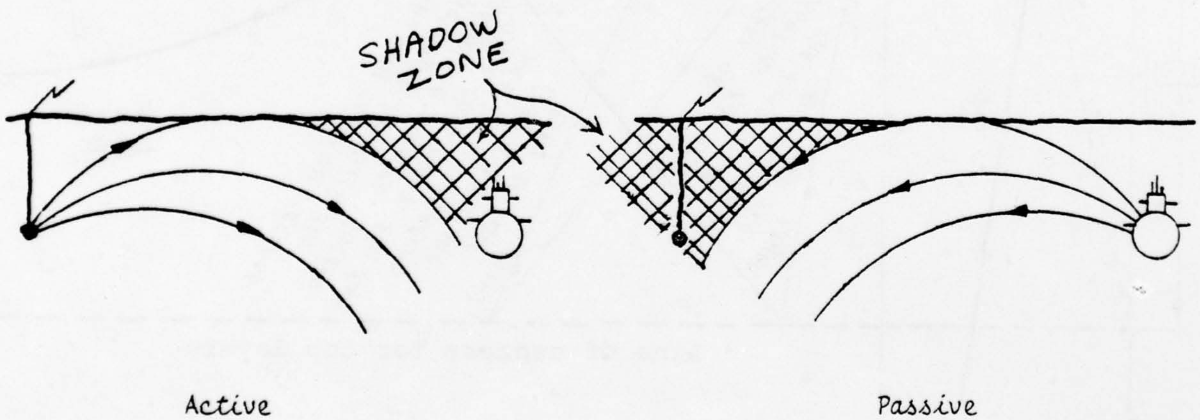
CONSTRUCTION OF RAY PATHS FOR TWO GRADIENTS



ACOUSTICAL RECIPROCITY

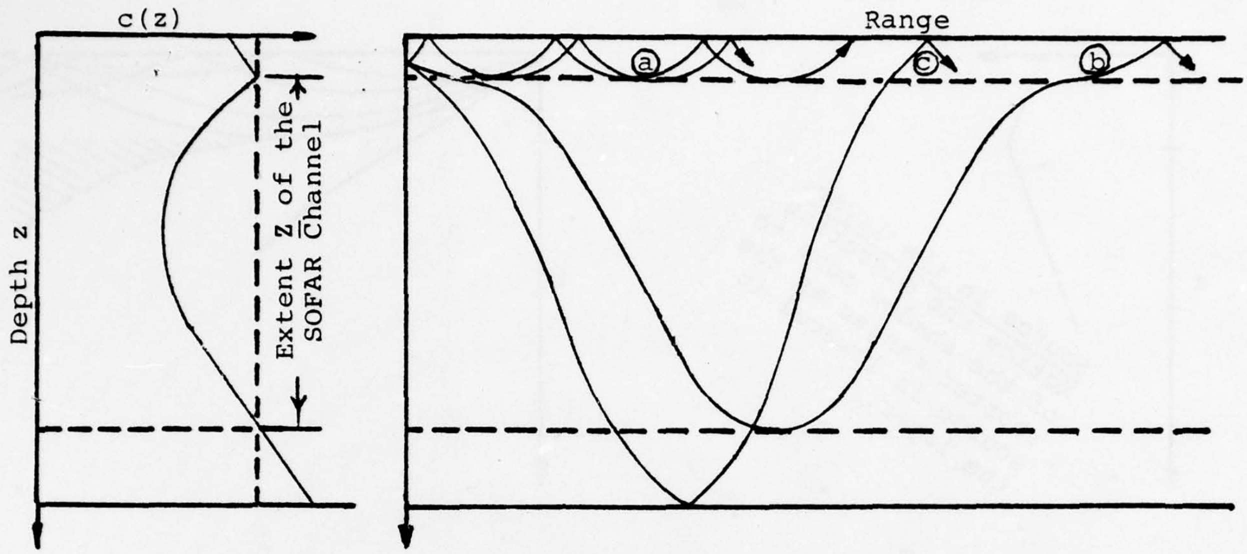


If the direction the sound travels over the propagation paths is reversed by exchanging the positions of the source and receiver, and the propagation conditions remain the same, then the transmission loss between source and receiver will remain the same.

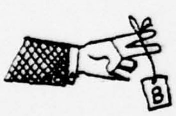


As a consequence, the transmission loss suffered by the signal in reaching the target from an active source will be identical with the transmission loss experienced by a signal of the same frequency (and duration) radiated from the target when it reaches a receiver.

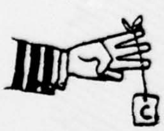
PROPAGATION PATHS FOR SHALLOW SOURCE AND RECEIVER



(a) Mixed Layer. The isothermal water at the ocean surface forms the mixed layer in which sound can be trapped. The continual reflection at the rough surface, refraction at the bottom of the layer, and leakage provide relatively large energy losses so that this path is useful for relatively short ranges.

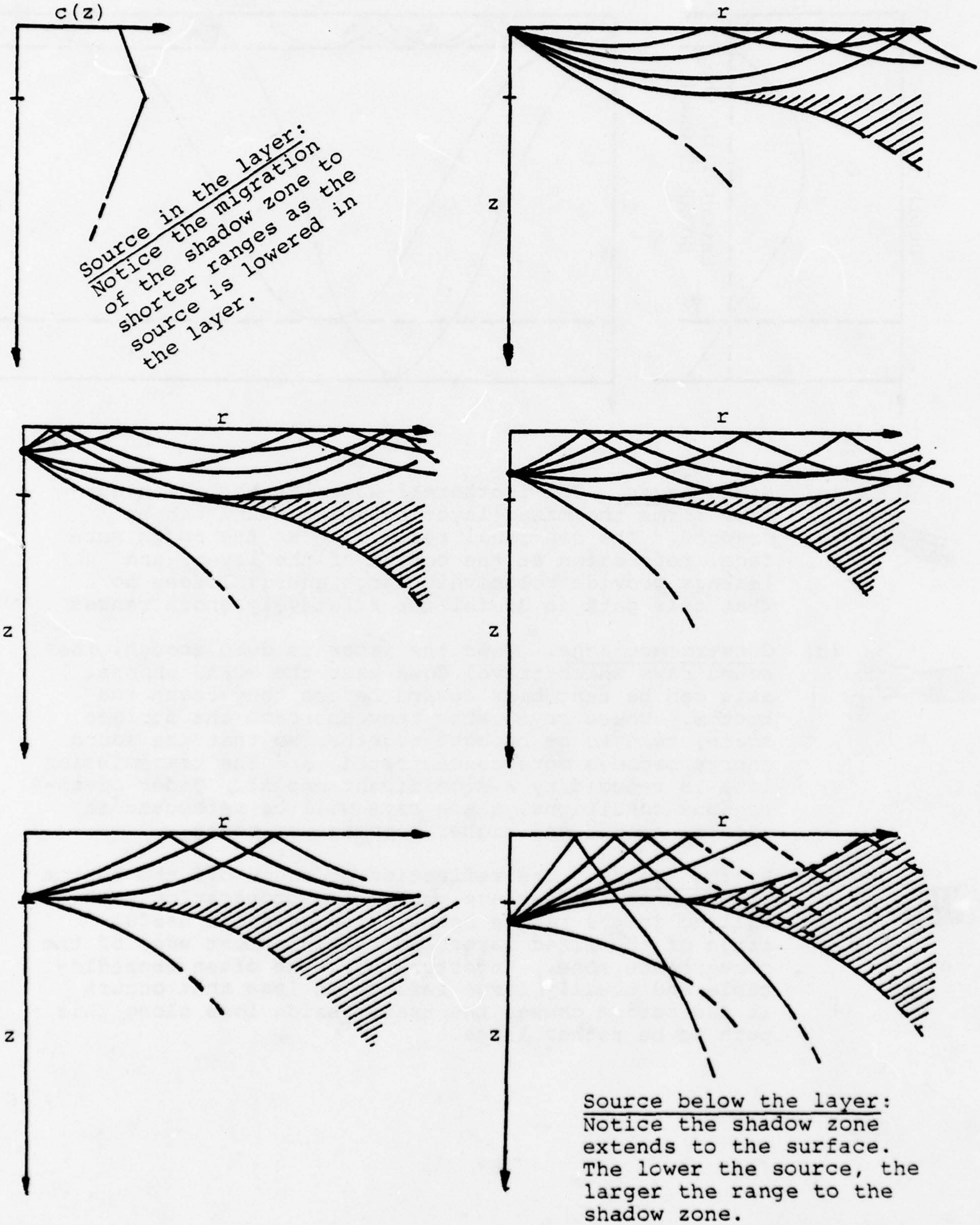


(b) Convergence Zone. When the water is deep enough, the sound rays which travel down past the SOFAR channel axis can be bent back upward before they reach the bottom. These rays, when they approach the surface again, tend to be brought together so that the sound energy becomes more concentrated, and the transmission loss is reduced by a significant amount. Under advantageous conditions, these rays will be refocused in second, third, and higher convergence zones.



(c) Bottom Bounce. The reflection of sound off the bottom and back to the surface provides a propagation path filling in the ranges between the greatest useful range of the mixed layer and the innermost edge of the convergence zone. Unfortunately, the often unpredictable and usually large reflection loss that occurs at the bottom causes the transmission loss along this path to be rather large.

RAYS AND A MIXED LAYER



LOSS FROM THE MIXED LAYER

When a ray reflects from the ocean surface, some of the energy is scattered into different directions and can propagate out of the mixed layer. This energy is thus lost from the channel, which increases the transmission loss. Additionally, energy can diffuse out of the bottom of the layer. These effects can be described in terms of the "loss per bounce". The number of bounces is the range divided by the skip distance r_s.

M. Schulkin, J. Acoust. Soc. Am. 44, 1152 - 1154 (1968) has developed from a vast amount of experimental data an equation over the frequency range 2kHz to 25kHz,

$$b = 1.04 \times (SS) \times \sqrt{f \text{ in kHz}} \, ,$$

where

b = loss per bounce in dB

SS = Sea State.

The equation is stated to be valid for $3 \lesssim b \lesssim 14$ dB/bounce.

Another empirical formula, resulting from the analysis of a number of experiments, has been obtained by W. F. Baker, J. Acoust. Soc. Am. 57, 1198 - 1200 (1975) --

$$b = 0.63 \times (1.4)^{SS} \times (f \text{ in kHz}) \, .$$

This equation is valid within a restricted range of parameters, given approximately by

$$2 < SS < 5$$

$$3 \text{ kHz} < f < 8 \text{ kHz}$$

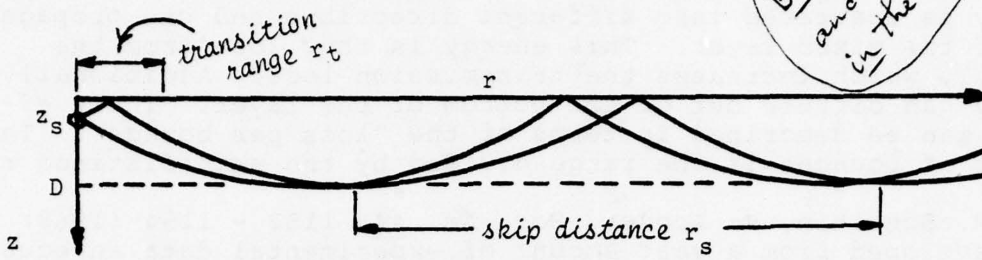
$$25 \text{ m} < \text{layer depth} < 70 \text{ m}$$

$$10 \text{ m} < \text{source depth} < 20 \text{ m}$$

Between 3kHz and 5kHz, these two equations are in fair agreement. Outside this frequency interval, it appears that Schulkin's formula is to be preferred.

TRANSMISSION LOSS IN THE MIXED LAYER

Both source and receiver in the layer



The depth of the mixed layer is designated by D.

The depth of the source in the layer is designated by z_s.

The <u>transition range</u> r_t is the distance the sound beam must travel until it effectively fills the channel. It is calculated from

$$r_t = 105\sqrt{\frac{D^2}{D - z_s}}\,, \qquad \text{(all distances are in meters)}$$

The <u>skip distance</u> r_s is the distance required for the rays which just graze the bottom of the layer to complete one full cycle from the layer bottom to the surface and back to the bottom. It is calculated from

$$r_s = 840\sqrt{D}\,. \qquad \text{(all distances are in meters)}$$

If the loss per bounce is designated by <u>b</u> and the energy loss resulting from absorption is designated by <u>a</u>, then the transmission loss for the mixed layer can be estimated by the following formulas:

$TL = 20 \log r + ar$	within transition range
$TL = 10 \log r_t + 10 \log r + ar + br/r_s$	beyond transition range

For ranges less than r_t the sound is spreading out <u>spherically</u> to fill the sound channel. For ranges greater than r_t, however, the sound has spread far enough vertically to be reflected from the surface and bent back upward from the bottom of the layer. Once this happens, then the sound is trapped and will spread <u>cylindrically</u>.

Notice that for $r > r_t$, the TL is reduced by having r_t as small as possible. From the formula for r_t, this would occur for $z_s = 0$. <u>HOWEVER</u>, if the source is within a <u>few wavelengths</u> of the surface, then the sound reflected from the surface can <u>drastically</u> interfere with that from the source so that the TL becomes larger than predicted.

IN GENERAL, THE TRANSITION RANGE r_t MUST BE CALCULATED WITH z_s BEING THE <u>LARGER</u> OF SOURCE AND RECEIVER DEPTHS.

REPRESENTATIVE PARAMETERS

Layer Depth D	Skip Distance r_s	Transition Range r_t* $z_s = 20m$	$z_s = 40m$	$z_s = 80m$
25 m	4200 m	1170 m	X	X
50	5940	960	1660 m	X
75	7270	1060	1330	X
100	8400	1170	1360	2350 m

*Assume the receiver is at a depth of 10 m.

SAMPLE CALCULATION OF A TRANSMISSION LOSS

Assumptions:

Depth of the mixed layer	=	40 m
Depth of the source	=	20 m (assume a shallow receiver)
Frequency of the sound	=	2 kHz
Sea state	=	3

We then determine

Skip distance	=	5300 m
Transition range	=	940 m
Attenuation constant	=	0.00015 dB/m
Loss per bounce	=	4.4 dB/bounce (Schulkin)

The transmission loss formulas are thus:

(a) For ranges less than the transition range,

$$TL = 20 \log r + (1.5 \times 10^{-4})r .$$

(b) For ranges greater than the transition range,

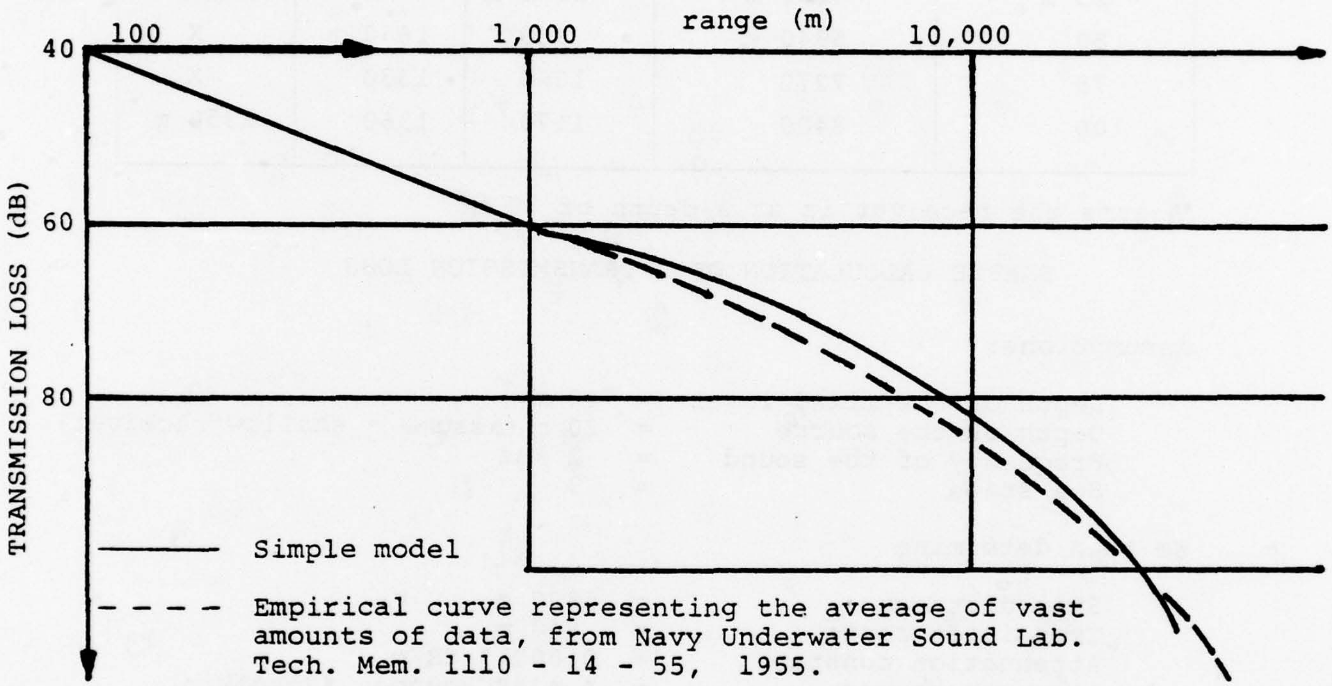
$$TL = 10 \log r + (9.8 \times 10^{-4})r + 30 .$$

The numerical values in these formulas are valid <u>only</u> for the parameters specified under Assumptions.

TRANSMISSION LOSS CURVE FOR SOURCE
AND RECEIVER IN THE MIXED LAYER

Parameters:

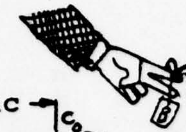
Depth of the mixed layer	=	40 m
Depth of the source	=	20 m (receiver is shallower)
Frequency of the sound	=	2 kHz
Sea state	=	3



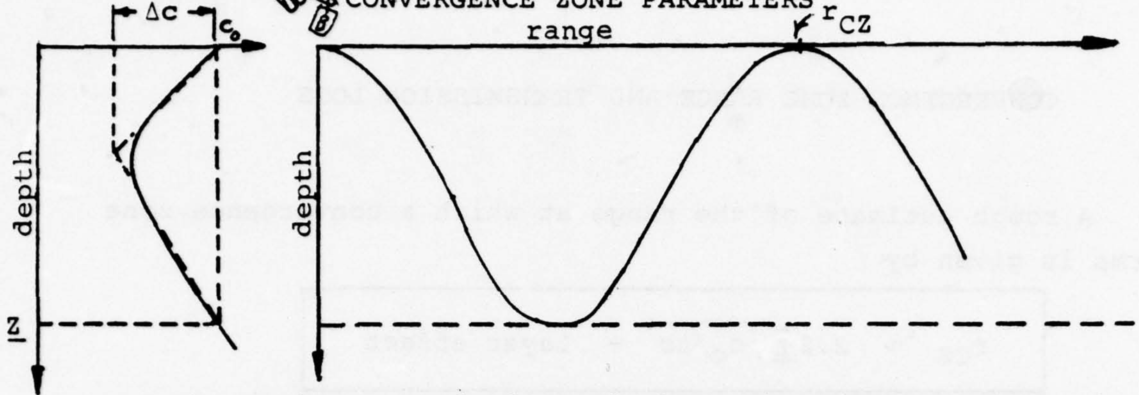
BUT: *These results can be radically changed by effects such as <u>Surface Interference</u> (which becomes important at lower sea states), steep gradients below the layer, and source and/or receiver near the top or bottom of the layer. Additionally, <u>if the frequency of the sound is too low, it will not be trapped in the mixed layer.</u>*

As a <u>very rough estimate</u>, <u>the frequency must be greater than about</u> $\boxed{f_c \sim (2 \times 10^5)/D^{3/2}}$ where D is in meters and the frequency is in Hz, for the sound to be well trapped in the layer and for the above model to apply. If $f < f_c$, it is probably best to assume spherical spreading with absorption.

CUTOFF FREQUENCY f_c



CONVERGENCE ZONE PARAMETERS



c_o = the maximum speed of sound above the SOFAR channel axis

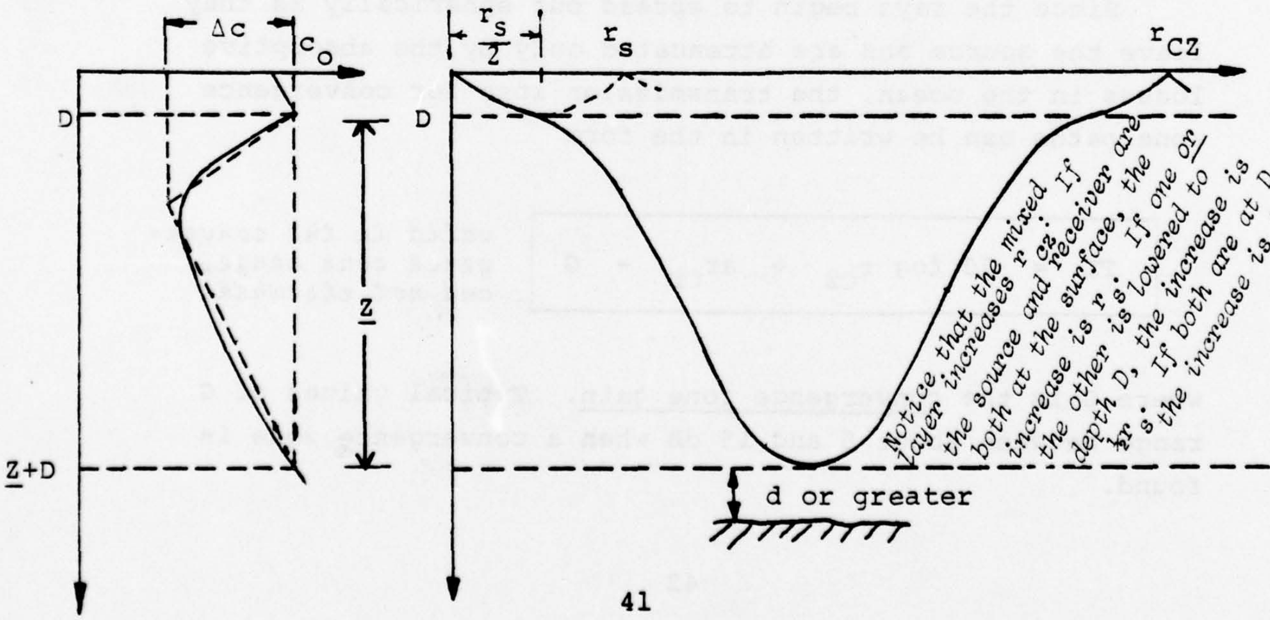
Δc = the difference between c_o and the speed of sound at the elbow

$\underline{Z}$ = the vertical distance between depths for which $c(z) = c_o$

r_{CZ} = the range to the convergence zone

Snell's Law reveals that a ray which is horizontal in the upper ocean where $c(z) = c_o$ will also be horizontal at a distance $\underline{Z}$ deeper, and a ray which is directed downward where $c(z) = c_o$ must penetrate below the distance $\underline{Z}$. To insure that enough rays are returned to the surface to focus at the convergence zone, the total depth of water must exceed $\underline{Z}$ or $\underline{Z}$ + D by an amount called the <u>depth excess</u>, d.

If there is no mixed layer, then c_o is associated with the surface and $\underline{Z}$ is measured from the surface. If there is a mixed layer, then c_o is associated with the bottom of the layer and $\underline{Z}$ is measured from the bottom of the layer.



CONVERGENCE ZONE RANGE AND TRANSMISSION LOSS

A rough estimate of the range at which a convergence zone forms is given by

$$\boxed{r_{CZ} \sim 2.8\underline{Z}\sqrt{c_o/\Delta c} + \text{Layer effect}}$$

If there is no mixed layer,
Layer effect $= 0$
If there is a mixed layer,
Layer effect ≈ 0 if source and receiver are both near the bottom of the layer
$\approx \frac{1}{2} r_s$ if one is near the top and other near the bottom of the layer
$\approx r_s$ if both are near the top of the layer.

The accuracy of the above expression depends on how well the constant-gradient approximations fit the true c(z) profile. As might be expected, when the elbow of the curve is very sharp, and the gradients above and below the elbow are relatively constant, then the prediction will be rather good. *The width of the convergence zone, over which the rays are appreciably focused, is normally about 10 per cent of the range to the zone, and* r_{CZ} *estimates the distance to the outer edge of the zone.*

Since the rays begin to spread out spherically as they leave the source and are attenuated only by the absorptive losses in the ocean, the transmission loss for convergence zone paths can be written in the form

$$\boxed{TL = 20 \log r_{CZ} + a r_{CZ} - G}$$

valid in the convergence zone range, and not elsewhere

where G is the convergence zone gain. Typical values of G range between about 5 and 15 dB when a convergence zone is found.

TYPICAL CONVERGENCE ZONE RANGES

North Pacific

Surface Temperature*		Minimum depth of CZ operation+		Range to first CZ	
50° F =	10° C	1270 ftm =	2320 m	47 kyd =	43 km
55	13	1610	2940	52	48
60	16	1900	3480	56	51
65	18	2150	3930	60	55
70	21	2400	4400	64	59
75	24	2600	4760	66	60
80	27	2800	5130	69	63

1 ftm = 1 fathom = 6 feet

Area	Surface Temperature		Minimum depth for CZ operation	Range to first CZ
N. Pacific	50°F =	10°C	1270 ftm	47 kyd
N. Atlantic	50	10	1050	46
Norwegian Sea	50	10	1680	53
Mediterranean	67	19	800	33

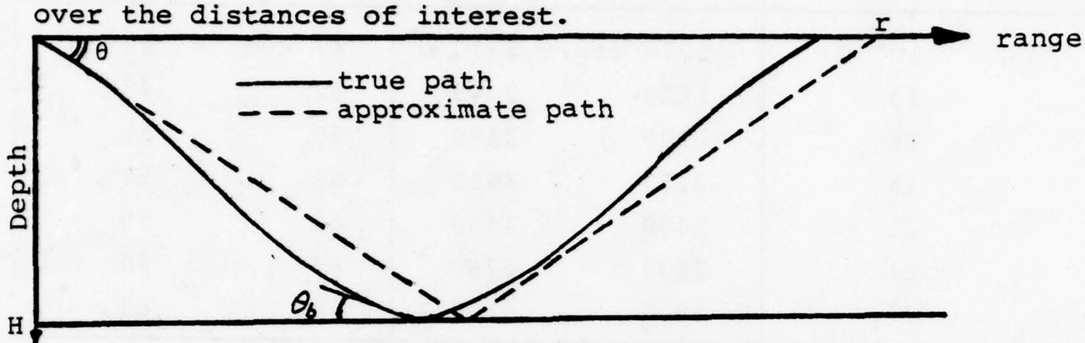
* Assumes mixed layer absent

\+ Allows for a 200 ftm depth excess

Ref. <u>Convergence Zone Range Slide Rule</u>, Naval Underwater Systems Center.

BOTTOM BOUNCE

For angles of depression greater than about 15° it is often possible to assume straight-line propagation of the rays over the distances of interest.



A straight-line ray bouncing off the bottom will return to the surface at a range

$$r = 2\,H/\tan\theta$$

where θ is the angle of depression.

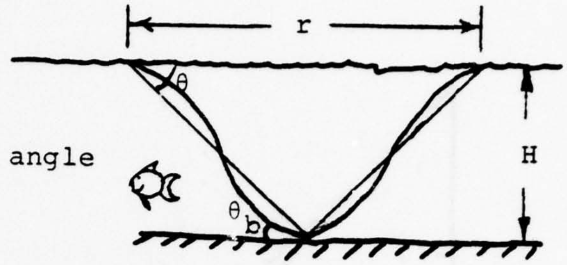
Since the distance travelled by the ray is $r/(\cos\theta)$, and since the rays spread out spherically and are attenuated only by the absorptive losses of the ocean and the reflection at the bottom, an approximate transmission loss can be written as

$$TL = 20\,\ell og\,\frac{r}{\cos\theta} + \frac{ar}{\cos\theta} + BL$$

where BL is the bottom loss in dB/bounce.

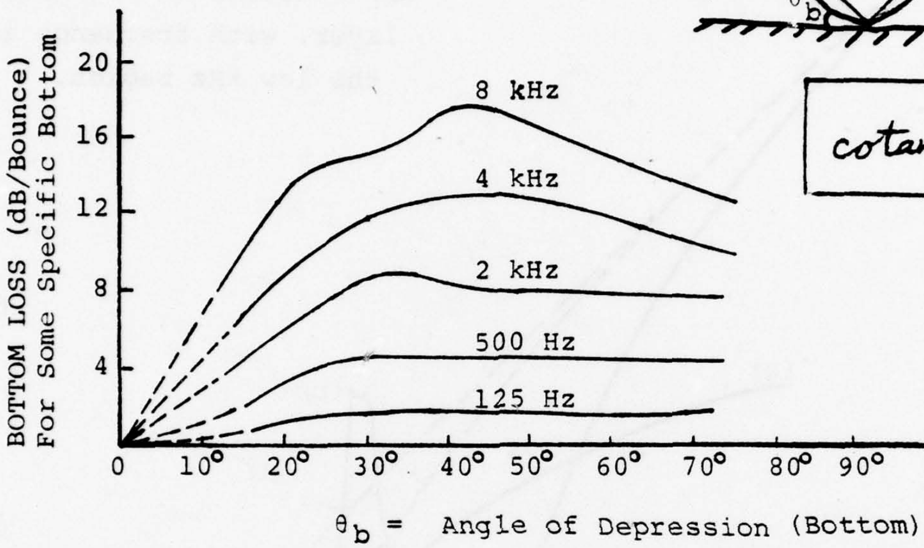
This formula can be trusted as long as the angle θ_b with which the ray strikes the bottom is not much different from the initial angle of depression θ. If θ_b is significantly different from θ, or goes to zero, then the ray path must be calculated more accurately by taking the curvature of the ray into account.

BOTTOM LOSS



$$\text{cotan}\,\theta = \frac{r}{2H}$$

<u>Depends</u> on frequency and depression angle



r/2H	1	1.19	1.43	1.73	2.14	2.75	3.73
θ	45°	40°	35°	30°	25°	20°	15°

<u>Depends</u> on bottom composition

Example: At 24 kHz and 10° depression angle

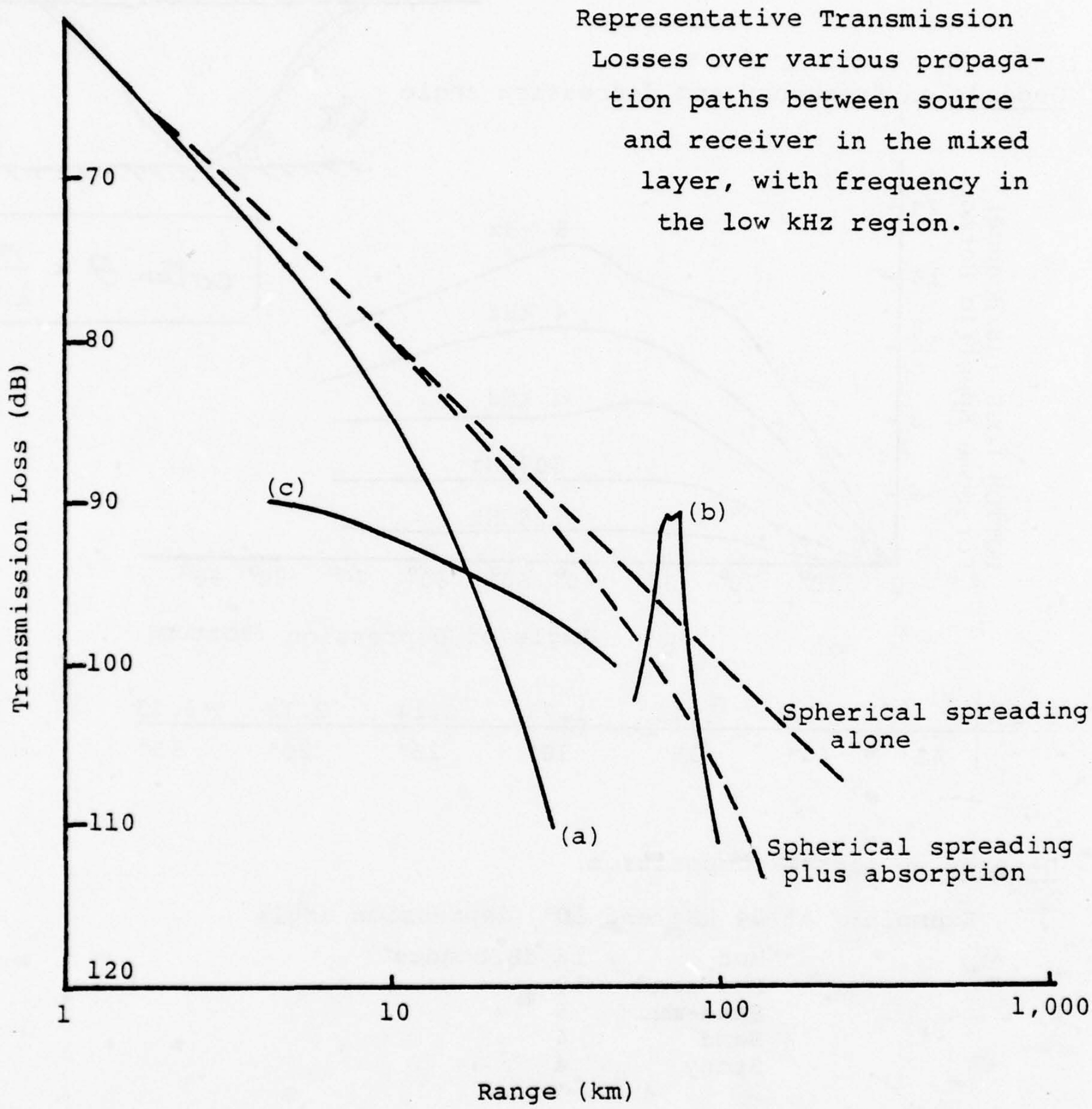
Mud	16 dB/bounce
Mud-sand	10
Sand-mud	6
Sand	4
Stony	4

<u>Depends</u> on surface roughness

Bottom loss goes up as ratio of roughness-to-wavelength increases.

Ref.

R.J. Urick, <u>Principles of Underwater Sound for Engineers</u>, McGraw Hill (1975).

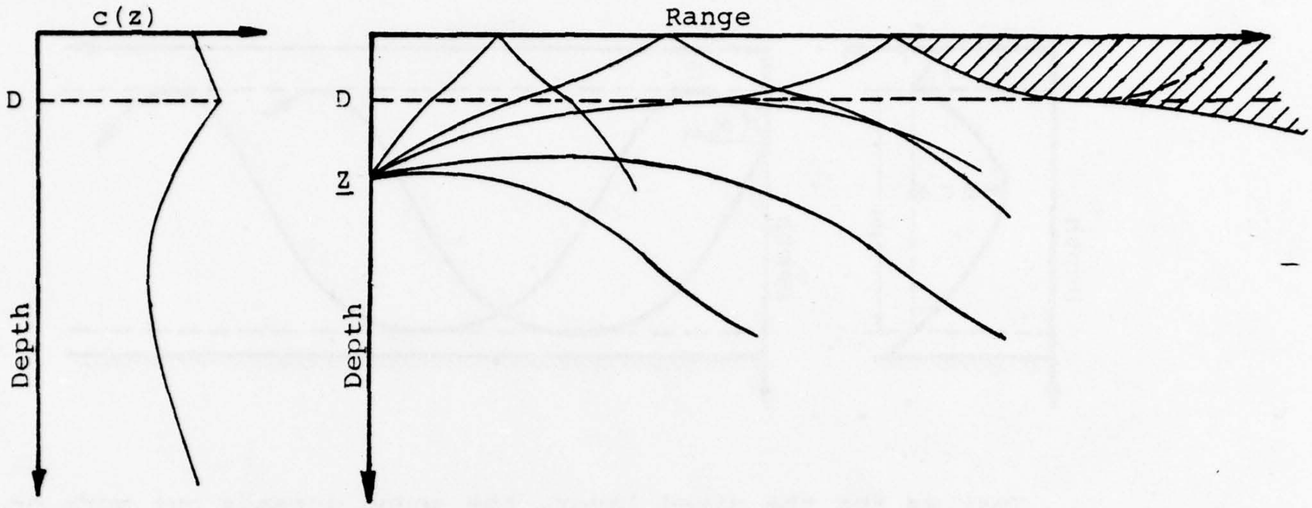


(a) Mixed Layer

(b) Convergence Zone

(c) Bottom Bounce

RELIABLE ACOUSTICAL PATH FOR DEEP SOURCE OR RECEIVER



Reliable Acoustical Path (RAP).

If the target is shallow and the sensor deep, or vice versa, there exists the possibility of a guaranteed acoustical path between the two. This path is useful to ranges beyond those attainable by transmission in the mixed layer. As the deeper of the two (sensor or target) is lowered farther and farther below the layer, the shadow zone indicated in the diagram moves out to larger ranges. If there is no mixed layer, then the leading edge of the shadow zone is formed by the downward-bending ray which just grazes the surface.

For positions of source and receiver such that there is direct ensonification, the rays can be considered to travel approximately in straight lines between the target and sensor. For this kind of propagation, the rays spread out spherically and the only losses are those from the absorption of sound in sea water. An approximate transmission loss formula is then

$$TL = 20 \log \frac{r}{\cos\theta} + \frac{ar}{\cos\theta}$$ *Invalid in the shadow zone*

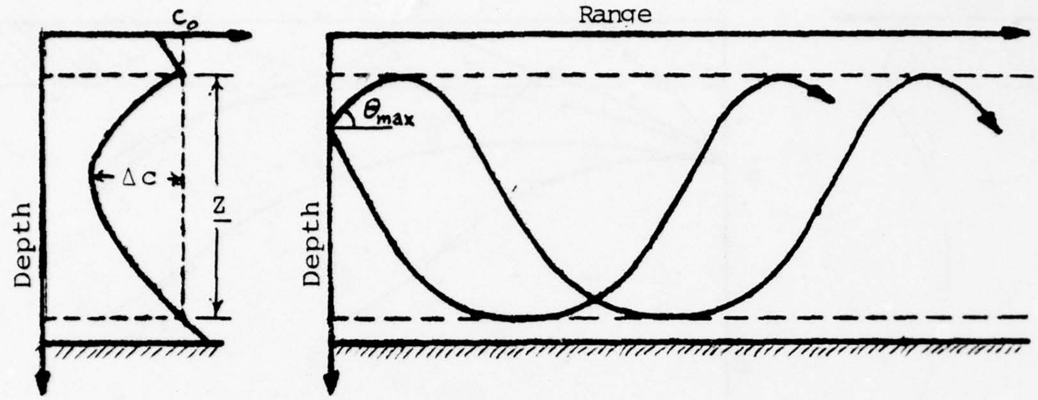
where θ is the angle of elevation of the rays between target and sensor and r in the (horizontal) range.

If the negative gradient (below the layer, if one exists) has magnitude g, then the range to the shadow zone at the surface is estimated by

$$1.4\sqrt{c_o(z-D)/g} + \tfrac{1}{2} r_s$$

where (z - D) is the depth below the bottom of the layer. If there is no layer, then D = 0 and there is no skip distance, so $r_s = 0$.

THE SOFAR CHANNEL



Just as for the mixed layer, the sound spreads out more or less spherically until the rays are bent back into the channel, after which the sound spreads cylindrically. Thus, beyond some transition range r_t' we should have a spreading term which goes as $10 \log r$. Those rays whose angles of inclination with respect to the SOFAR channel axis are large enough that they reflect from the ocean surface or bottom will suffer reflection losses and scattering losses and eventually be so weak compared to the trapped rays that they can be neglected. Thus, for propagation in the channel, there are no losses except absorption, so that the transmission loss has the simple form

$$TL = 10 \log r + 10 \log r_t' + ar \qquad \text{for ranges beyond } r_t'.$$

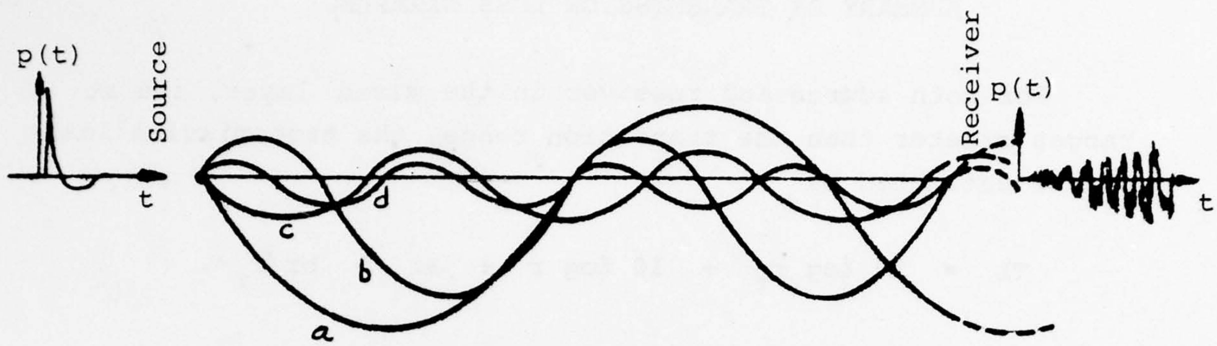
The transition range depends on how deep the sound source is in the channel and the vertical extent of the channel. For sources placed reasonably far down in the channel, the transition range is less than the convergence zone range r_{CZ}. Notice that the rays cycle up and down in the channel, accomplishing one cycle over an interval very similar to the convergence zone range,

$$r_t' \sim \frac{Z}{2 \tan \theta_{max}} .$$

θ_{max} is determined for source and receiver; the smaller value is used for finding r_t'.

The above transmission loss formula cannot be trusted at moderate ranges because of the spotty ensonification of the channel. Beyond several convergence zones the acoustic energy tends to be more uniformly distributed over the vertical extent of the channel and the above formula more trustworthy. However, the farther the source and receiver are above or below the channel axis, the larger the transition range becomes and the greater the distance that is required for the energy to become distributed more or less uniformly.

TIME STRETCHING IN THE SOFAR CHANNEL



An interesting fact often observed in the SOFAR channel is that transient signals are "stretched" in time as they propagate to large ranges. This arises because of the variation in the speed of sound over the vertical extent of the channel. For the SOFAR channels commonly encountered in the northern hemisphere, the speed of sound increases sufficiently rapidly with distance above or below the channel axis that sound traveling over those rays which swing over large vertical distances, like a and b in the above sketch, reaches the receiver at times earlier than sound traveling over the more direct paths c, and d. Thus, the sound that travels straight down the axis of the channel arrives at the receiver last. This leads to the characteristic blossoming of the received signal from an explosive charge, the sound first being very weak, and then growing in volume until there is a sudden cutoff of sound as the last signals come over the slowest paths.

The exact details of these effects depend greatly on the character of the speed of sound profile: If the sound speed does not increase sufficiently rapidly above and below the axis, for example, the time stretching may not occur, or may even be reversed, with the sound traveling straight down the axis arriving first. Additionally, the exact location of the source and receiver with respect to the channel axis is very important with respect to the envelope of the received signal and the amount of time stretching encountered. For the typical profiles in the northern hemisphere, the increased duration Δt of the received signal over that sent can be estimated by

$$\frac{\Delta t}{t} = \frac{1}{3}\frac{\Delta c}{c}$$

if source and receiver are on the channel axis, and less if they are not. The quantity t is the total time of flight between source and receiver and Δc is the maximum variation in the speed of sound between the upper and lower boundaries of the channel indicated in the previous page. Typically, this leads to about 9 seconds of stretching for each 1000 nautical miles the signal has travelled.

SUMMARY OF TRANSMISSION LOSS EXAMPLES

For both source and receiver in the mixed layer, and at ranges greater than the transition range, the transmission loss can be estimated by

$$TL = 10 \log r_t + 10 \log r + ar + br/r_s .$$

For a convergence zone detection, we can write

$$TL = 20 \log r_{CZ} + ar_{CZ} - G .$$

If the propagation path is bottom bounce, and the angle of depression of the sound beam in that mode is sufficiently large, then the simple geometrical model gives

$$TL = 20 \log \frac{r}{\cos \theta} + \frac{ar}{\cos \theta} + BL .$$

For propagation between a deep point and a shallow point over the reliable acoustical path, an approximate transmission loss formula is

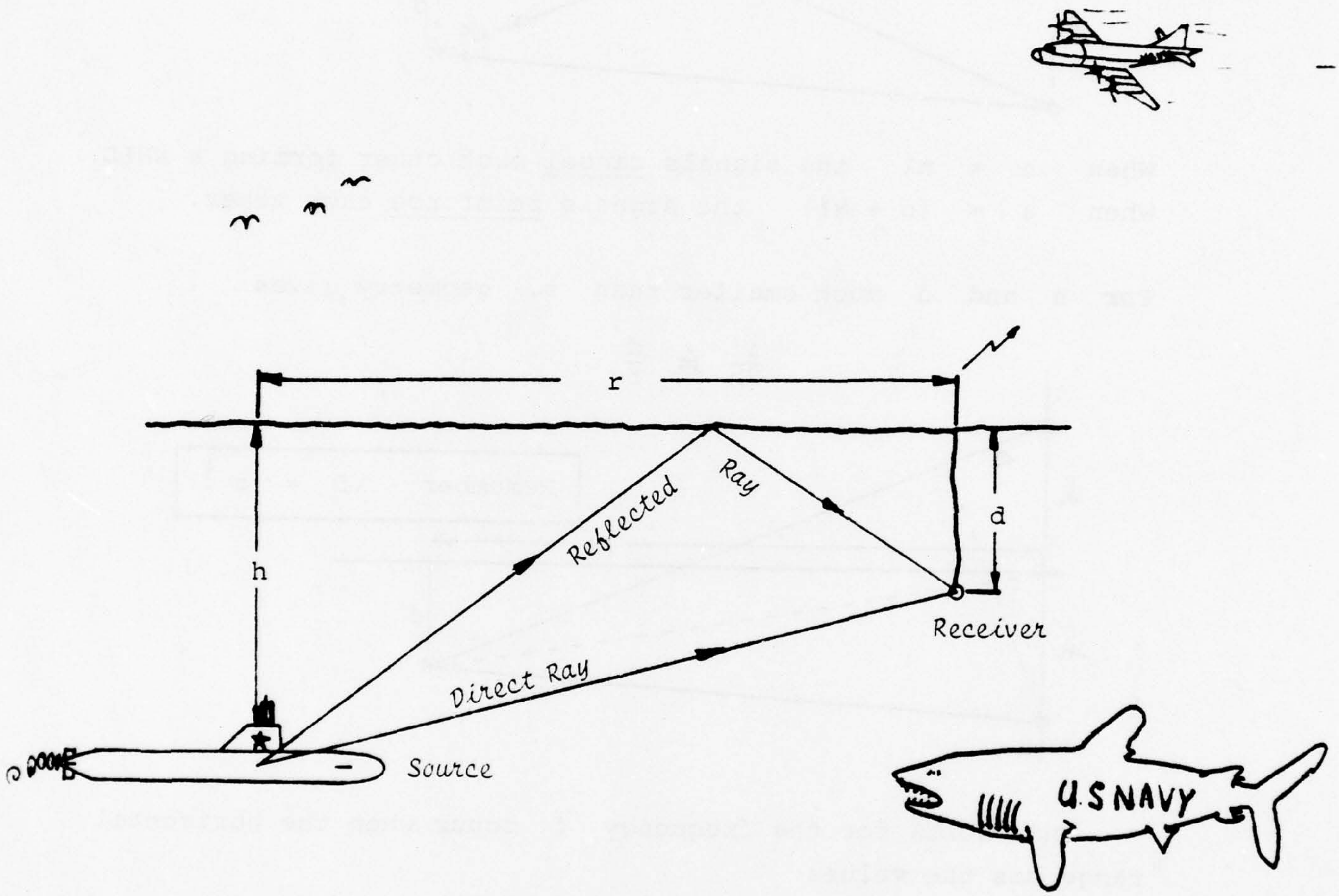
$$TL = 20 \log \frac{r}{\cos \theta} + \frac{ar}{\cos \theta} .$$

Finally, for the propagation of sound trapped in the SOFAR channel, if the range is greater than a few convergence zone separations, the transmission loss can be estimated by

$$TL = 10 \log r + 10 \log r_t' + ar .$$

REMEMBER THAT r_t IS CALCULATED FROM THE GREATER OF SOURCE AND RECEIVER DEPTHS.
ALSO, RECALL THAT r_t' COMES FROM THE SMALLER OF θ_{max} FOR SOURCE AND RECEIVER.

SURFACE INTERFERENCE



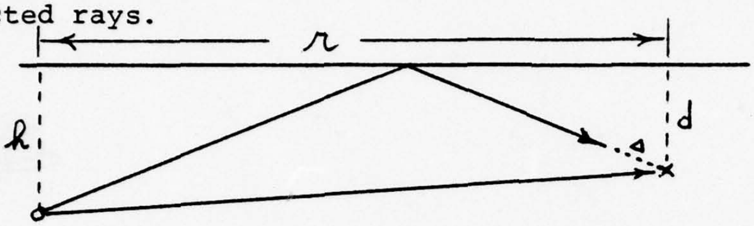
h = Source Depth

d = Receiver Depth

r = Horizontal Range

INTERFERENCE NULLS IN ISOVELOCITY WATER

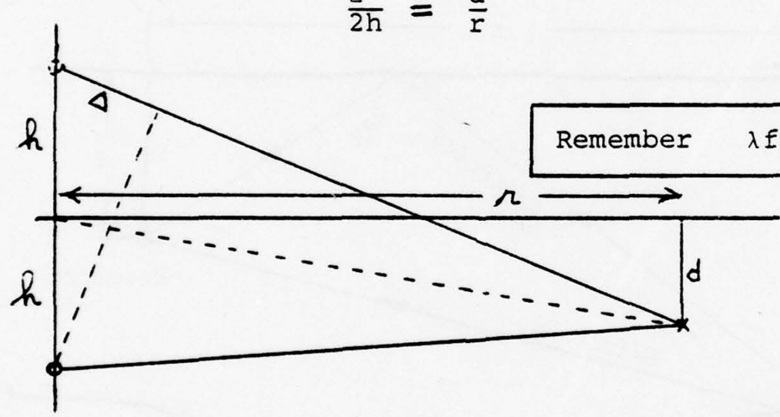
The following figures illustrate the geometry, showing explicitly the path length difference Δ between direct and reflected rays.



When $\Delta = n\lambda$ the signals cancel each other forming a NULL.
When $\Delta = (n + \frac{1}{2})\lambda$ the signals reinforce each other.

For h and d much smaller than r, geometry gives

$$\frac{\Delta}{2h} \doteq \frac{d}{r}$$



Remember $\lambda f = c$!

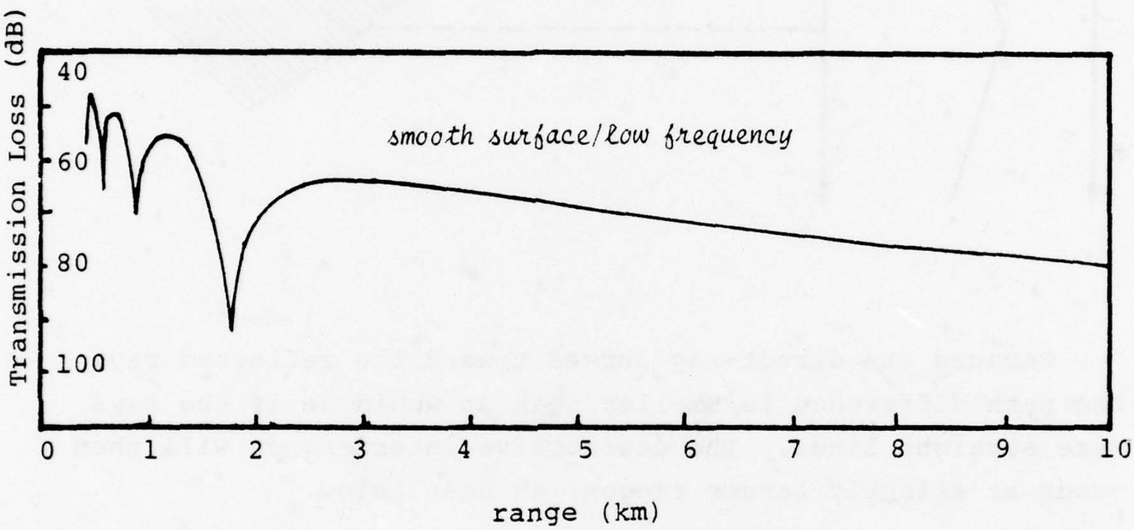
Thus NULLS for the frequency f occur when the horizontal range has the values

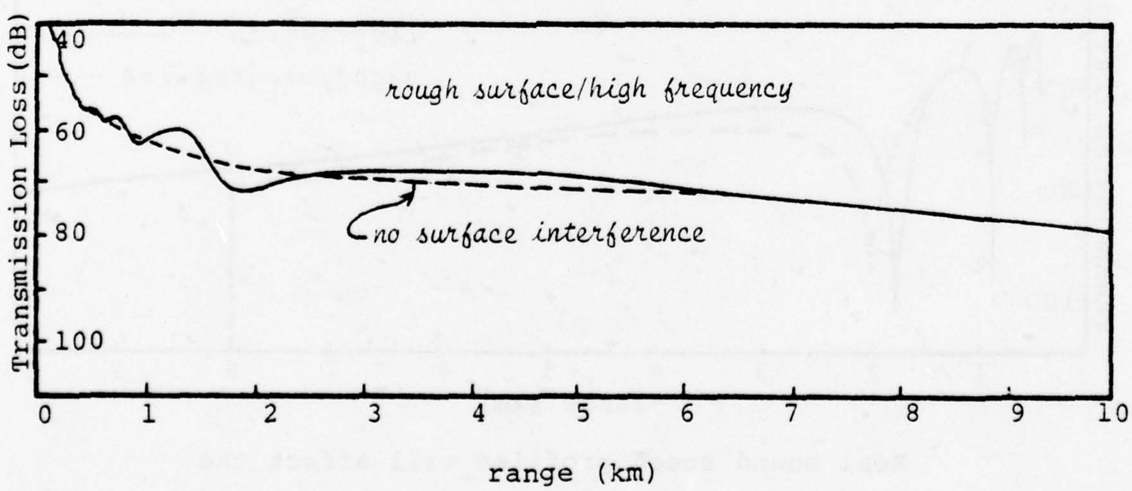
$$r_n = \frac{1}{n} 2hd \frac{f}{c} \qquad n = 1, 2, 3, \ldots.$$

NULLS

IMPORTANCE OF SURFACE SMOOTHNESS

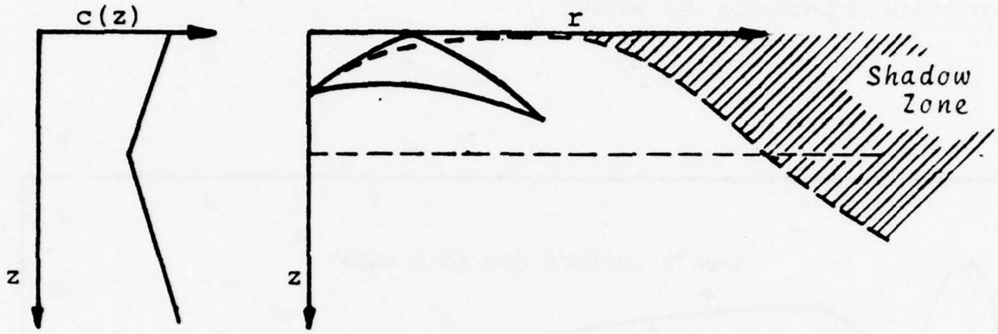
The rays will interfere and give a strong pattern if the surface is smooth and the frequency of the sound is low. Usually, f must be below <u>about</u> 1 kHz. If the surface is rough, then the reflected ray is scattered into many directions and interference is weak.



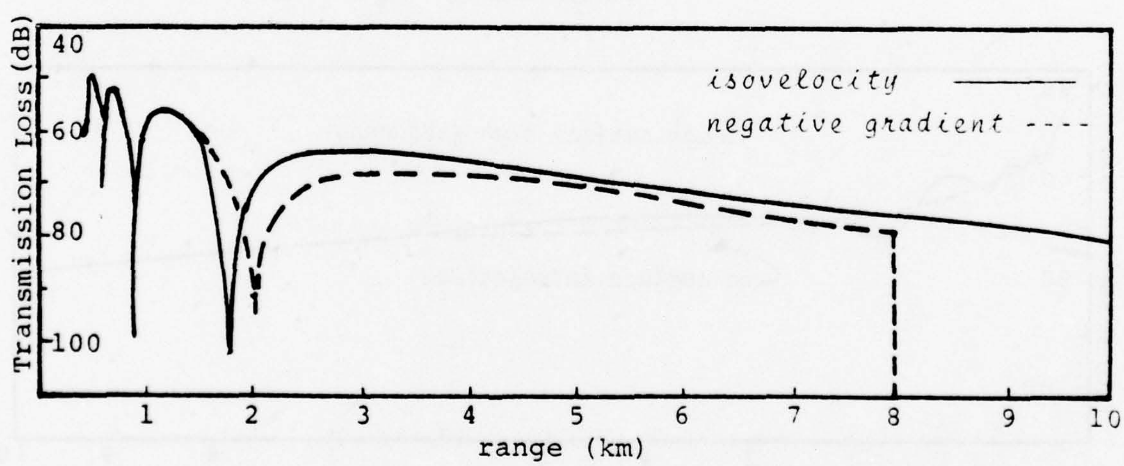


SURFACE INTERFERENCE IN AN ISOGRADIENT PROFILE

The bending of the rays can change the surface interference pattern. For example, in a negative-gradient layer the following ray paths may be observed:



Because the direct ray curves toward the reflected ray, the path difference is smaller than it would be if the rays were straight lines. The destructive interference will then occur at slightly larger ranges, as seen below.

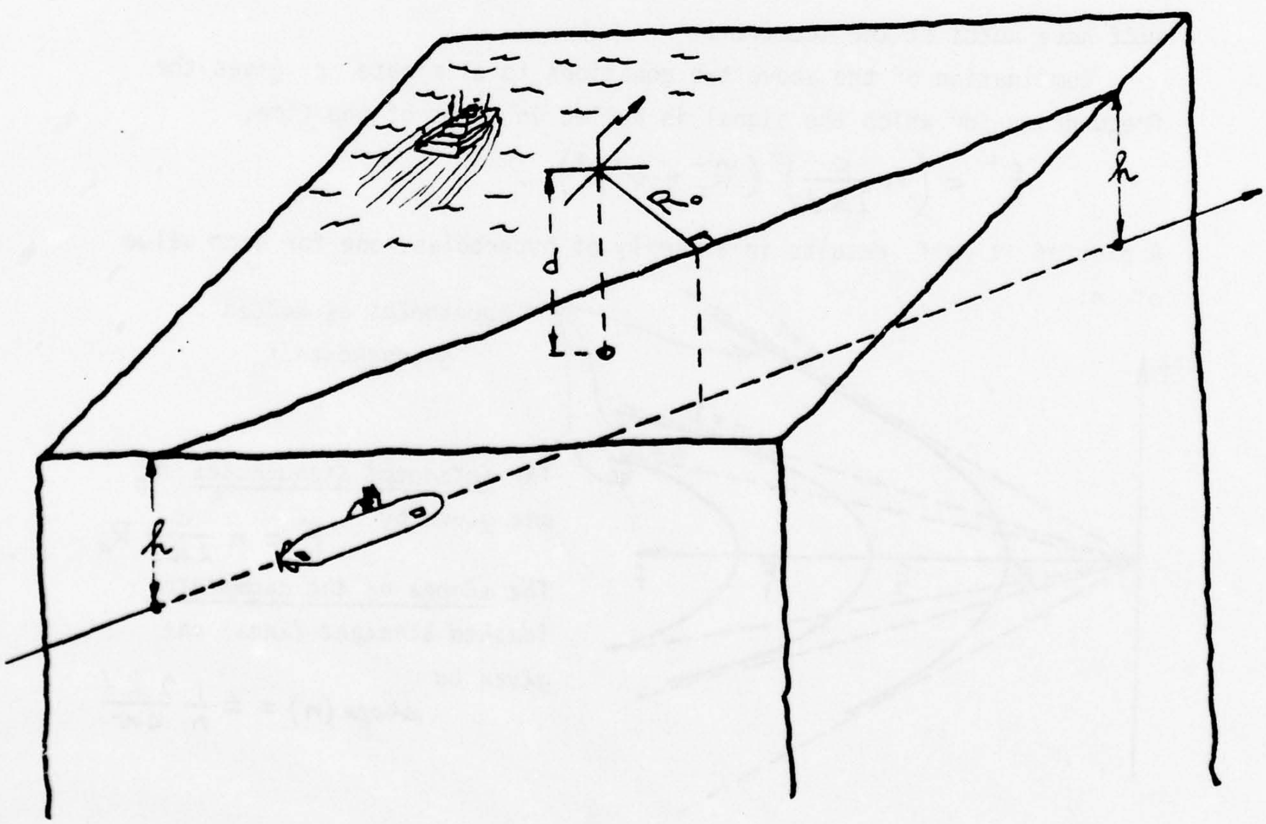


Real sound speed profiles will affect the surface interference effect chiefly at larger distances. (Notice the shadow zone predicted at 8 km.)

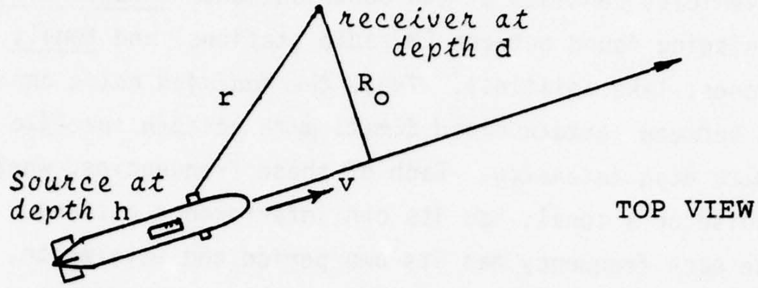
A USE OF SURFACE INTERFERENCE

As will be developed in more detail later, the noise radiated by underwater vehicles consists of two contributions: broadband noise (similar to the hissing found between FM radio stations) and tonals (single frequency tones, like whistles). *Thus, the radiated noise consists of all frequencies between certain broad limits with certain specific frequencies occurring with high intensity.* Each of these frequencies, whether of broadband noise or a tonal, has its own interference pattern.

Because each frequency has its own period and wavelength, each will have a different phase-delay for propagation over the direct and surface-reflected paths. As a result, the ranges between source and receiver for which the surface-reflected signal tends to cancel the direct signal will be different for each different frequency. *Thus, the ranges at which different frequencies will have nulls at the receiver will be different.* This means that as the range between source and receiver changes, the frequencies which have nulls at the receiver will also change.



It is possible to use this information to determine the depth h of the source and distance of closest approach R_0 of the source to the receiver. Let the source be travelling with speed v in a straight line:



The range is given by

$$r^2 = R_0^2 + v^2 t^2$$

where at $t = 0$ we have $r = R_0$, the distance of closest approach.

For given source depth h, *receiver depth* d, *and range* r, *those frequencies satisfying the equation*

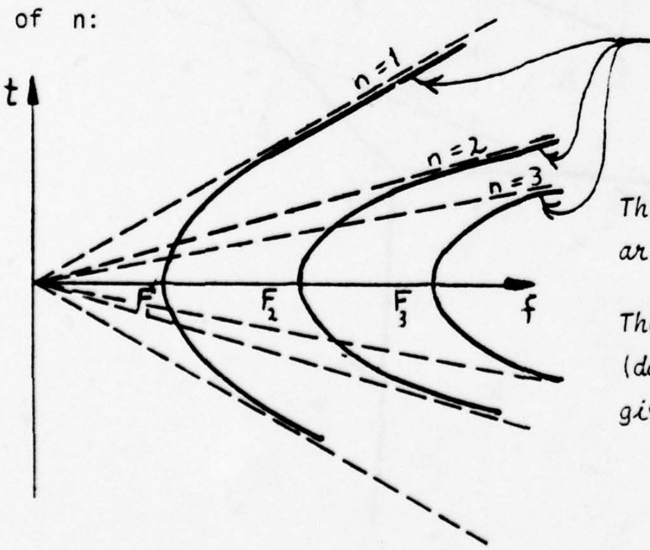
$$f_n = n \frac{c}{2hd} r \qquad r \gg h \text{ and } d \qquad n = 1, 2, 3, \ldots$$

will have nulls at the receiver.

Combination of the above two equations to eliminate r gives the frequencies for which the signal is nulled in terms of the time,

$$f_n^2 = \left(n \frac{c}{2hd}\right)^2 \left(R_0^2 + v^2 t^2\right) .$$

A plot of t vs f results in a family of hyperbolas, one for each value of n:



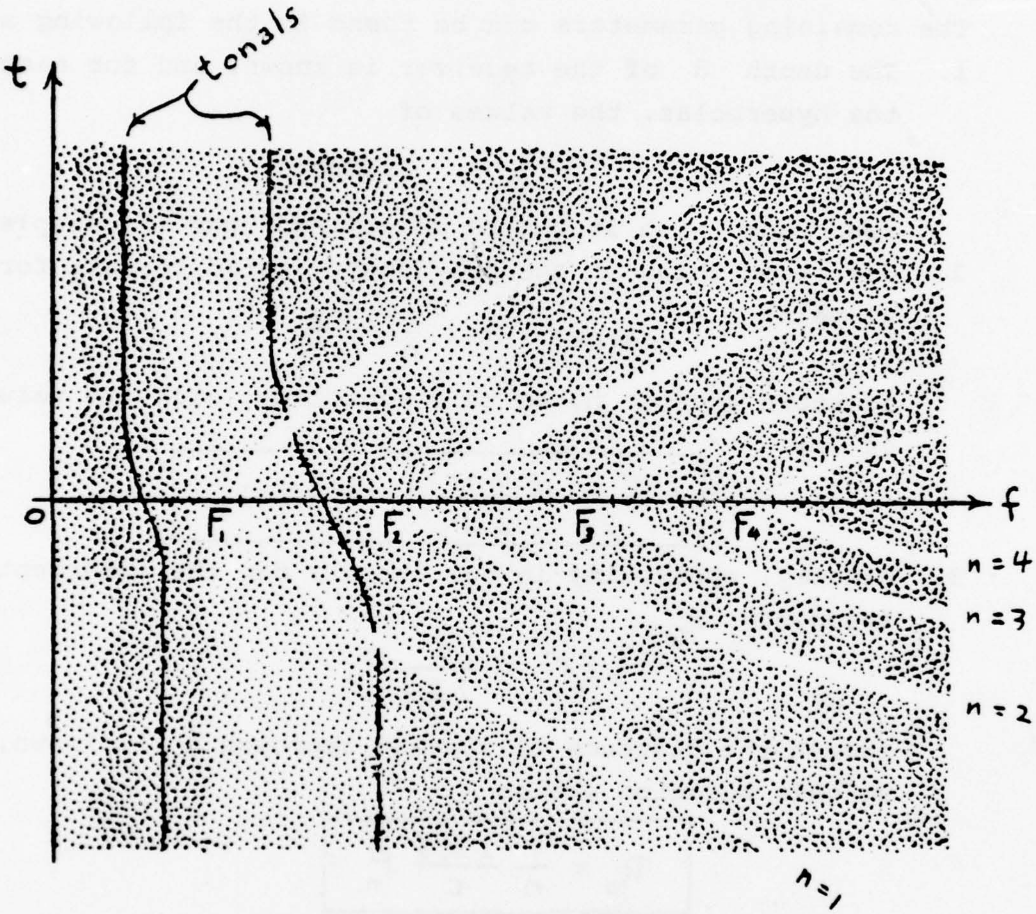
The *intercept frequencies* F_n are given by

$$F_n = n \frac{c}{2hd} R_0$$

The *slopes of the asymptotes* (*dashed straight lines*) are given by

$$\text{slope}(n) = \pm \frac{1}{n} \frac{2hd}{cv}$$

When this is displayed as in a LOFAR gram, the nulled frequencies appear as white curves (unburnt by the electric sparker that marks the paper) against the darkened regions which have been burnt by the sparker. These darkened regions correspond to sound of sufficient strength received from the source to activate the sparker. *Any distinct tonal will show the effects of Doppler shift, being higher in frequency when the source is approaching the receiver, and dropping to a lower frequency after the source has passed the receiver and is receding from it.* The resultant display is shown below:



The mottled background corresponds to the broadband noise emitted by the source. Two tonals are shown, one lying below frequency F_1, another lying between F_1 and F_2. Notice the shifts in frequency between very early times and later times after the source has passed the receiver. The white hyperbolas are described by the frequency F_n which is nulled at the distance of closest approach. By inspection of the display, it is possible to identify the value of n that is to be associated with each hyperbola, and the slope of each asymptote of the hyperbolas can be directly measured.

As will be discussed in more detail later, the Doppler shift of the tonals can be used to determine the speed v with which the source is moving from the equation

$$\boxed{v = \frac{1}{2} \frac{\Delta f}{f} c}$$

where Δf is the total change of frequency of a tonal whose frequency at the time of closest approach is f.

The remaining parameters can be found in the following steps:

1. The depth d of the receiver is known, and for each of the hyperbolas, the values of

   $n \qquad F_n \qquad \text{slope}(n)$

   associated with it can be determined from the display.
2. Now, everything in the previously-stated formula for slope(n)

   $$\text{slope}(n) = \pm \frac{1}{n} \frac{2hd}{cv}$$

   is known except h, so this equation yields the value of h:

   $$\boxed{h = n \frac{cv}{2d} \left|\text{slope}(n)\right|}$$
3. Finally, everything in the formula for the intercept frequency F_n

   $$F_n = n \frac{c}{2hd} R_o$$

   except the distance of closest approach R_o is known, so this equation yields R_o:

   $$\boxed{R_o = \frac{1}{n} \frac{2hd}{c} F_n}$$

Thus, at low frequencies for which there is an observable surface interference effect, the depth h, speed v, and distance of closest approach R_o of a source to a receiver of known depth d can all be determined from the observed Doppler shift of the tonals and the time-dependence of the frequencies which are nulled at the receiver.

NORMAL MODES (ignoring losses)

A more exact approach to the propagation of sound in the ocean is Normal Mode Theory. If the mathematical difficulties of this approach can be overcome, it yields better values in certain situations for the Transmission Loss than can be obtained using Ray Theory.

Normal Mode Theory yields a sum of individual solutions to the acoustical wave equation, each called a normal mode, which must be combined to obtain a complete expression for the sound field:

$$p = p_1 + p_2 + \cdots + p_n + \cdots + p_N = \sum_{n=1}^{N} p_n$$

FOR LARGE DISTANCES r FROM THE SOURCE OF SOUND ---

Each one of these normal modes has its own behavior,

$$\boxed{p_n = Z_n(z)\,\frac{A_n}{\sqrt{r}}\,\sin\left[2\pi f\left(t - \frac{r}{c_n}\right)\right]} \qquad r >> 1$$

This part looks just like a cylindrical wave traveling outward with its amplitude falling off as $1/\sqrt{r}$. Notice however that it has its own special phase speed c_n .

This part is an amplitude factor which changes with depth. Notice that each normal mode has its own special depth-dependent amplitude factor.

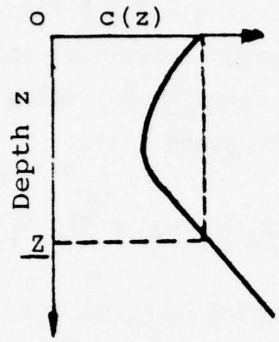
These special factors Z_n are called normalized eigenfunctions. They are found by solving the wave equation and specifying the behavior of the pressure at the ocean surface and bottom. The solution for each Z_n also gives a required value for c_n. Both the depth dependence of Z_n and the value of c_n depend on the frequency of the sound waves.

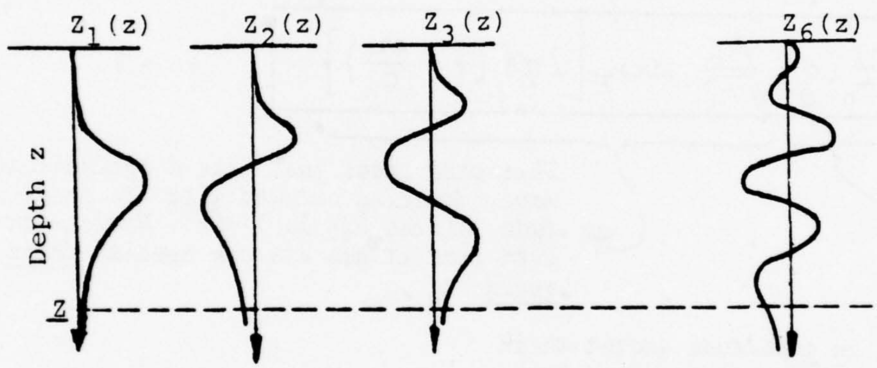
This kind of a solution can be obtained whenever the speed of sound profile c(z) *is shaped so that the sound can be trapped in a channel. The trapping of sound in the mixed layer is one such example, and the propagation of sound to long ranges in the SOFAR channel is another.*

If the depth of the source is z_0, and the radiated wave has effective pressure amplitude P(1) at 1 meter from the center of the source, then (complicated) mathematical analysis beyond the scope of this course shows that

$$\boxed{A_n = \sqrt{2}\, P(1)\sqrt{c_n/f}\; Z_n(z_0)} \qquad \text{no losses}$$

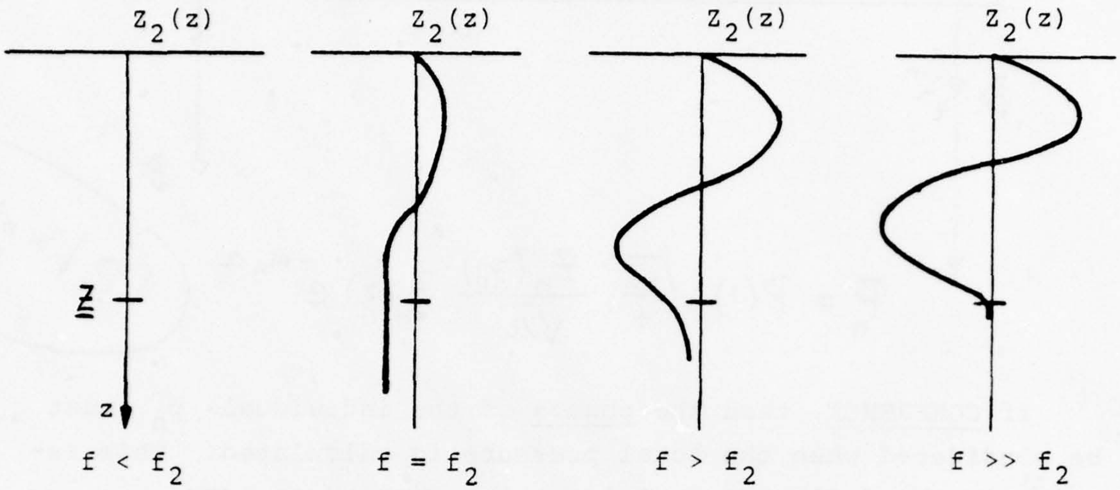
NORMAL MODES FOR A CHANNEL





The above sketches represent the normalized depth eigenfunctions of the first three and sixth normal modes for sound trapped in the SOFAR channel. These sketches show that the greatest part of the pressure field is found in the channel, but the long "tails" which extend to depths below the bottom $\underline{Z}$ of the channel show that there is sound ensonification at depths greater than those predicted by ray paths.

CUTOFF FREQUENCY AND LOSSES



Each normal mode has its own <u>*cutoff frequency*</u> f_n. *If the sound has a frequency below that of cutoff* <u>*for a particular*</u> *mode, then that mode cannot be excited to carry sound energy in the channel.*

If the frequency of the sound is slowly increased with time, the above sketches show how a normal mode can be excited. Just above cutoff, the normal mode has a very long tail which goes to great depths beyond the edges of the channel. As the frequency increases further above cutoff, the tail gets shorter and shorter, until at frequencies well above f_n there is very little sound existing outside the channel.

This means that for a specific frequency f, *sound can be trapped and sent to large distances* <u>*only*</u> *in those normal modes whose cutoff frequencies* f_n *are less than the frequency of the sound. Those whose cutoff frequencies exceed the frequency of the sound cannot be excited at all.*

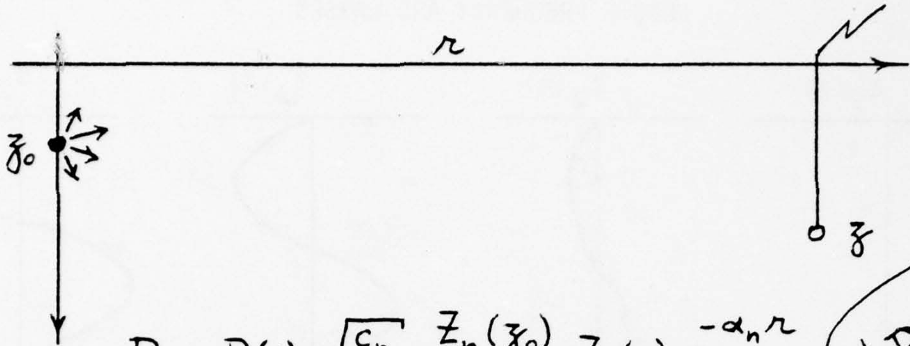
The larger the tail extending out of the channel, the more energy can be lost from the sound in the channel. This means that higher modes (those with higher cutoff frequencies) will attenuate faster and be less significant at great distances. This effect is sometimes called <u>mode stripping</u>.

These energy losses for each mode can be handled by specifying an absorption coefficient α_n for each normal mode. Then, we have

$$\boxed{p_n = \sqrt{2}\left\{P(1)\sqrt{\frac{c_n}{f}}\,\frac{Z_n(z_0)}{\sqrt{r}}\,Z_n(z)\,e^{-\alpha_n r}\right\}\sin\left[2\pi f\left(t-\frac{r}{c_n}\right)\right]}$$

where α_n increases with increasing n. The magnitude $|P_n|$ of $P_n = \{\ \}$ is the <u>effective pressure amplitude</u> of p_n.

TRANSMISSION LOSS (NORMAL MODES)



$$P_n = P(1)\sqrt{\frac{c_n}{f}}\,\frac{Z_n(z_0)}{\sqrt{r}}\,Z_n(z)\,e^{-\alpha_n r}$$

$|P_n|$ = effective amplitude of P_n

If COHERENCE, then the phases of the individual p_n must be considered when the total pressure is calculated. This results in the following form for the Transmission Loss:

$$TL = -20\log\left[\sqrt{\left(\sum_n P_n \sin\frac{2\pi f r}{c_n}\right)^2 + \left(\sum_n P_n \cos\frac{2\pi f r}{c_n}\right)^2}\Big/ P(1)\right]$$

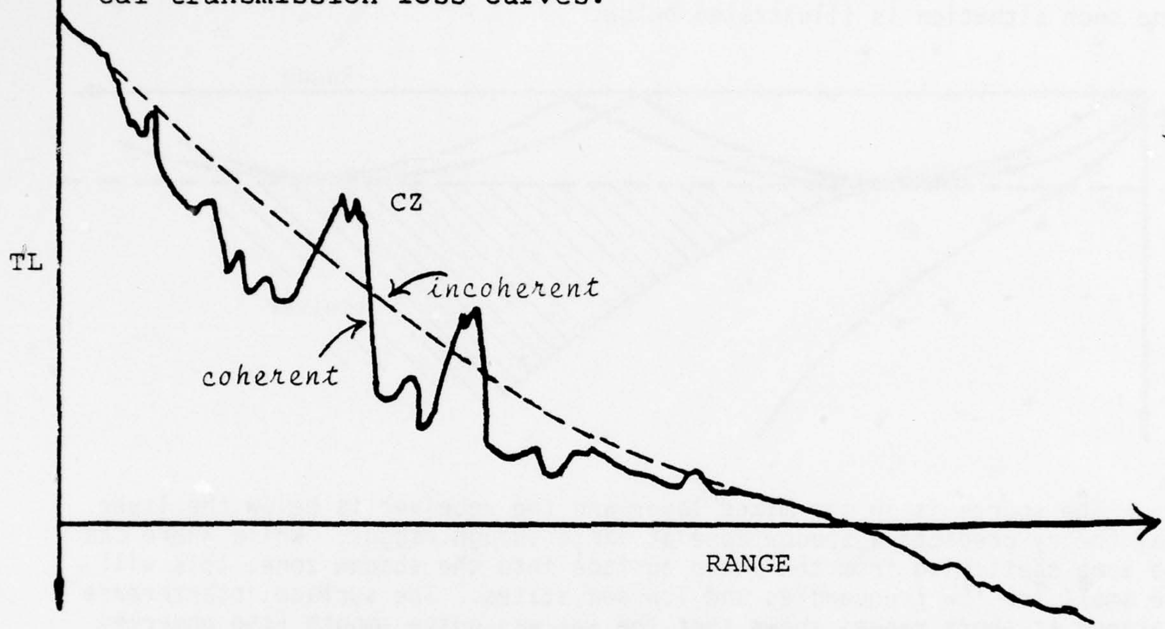
If INCOHERENCE, then the intensities of the individual modes will add, giving a Transmission Loss

$$TL = -20\log\left[\sqrt{\sum_n P_n^2}\Big/ P(1)\right]$$

Each type of combination is favored by its own special conditions, as follows:

COHERENCE	INCOHERENCE
low frequency	high frequency
short ranges	long ranges
homogeneous water	inhomogeneous water
smooth boundaries	rough boundaries

The differences in effect between COHERENT and INCOHERENT combination of the normal modes are illustrated below in typical transmission loss curves.



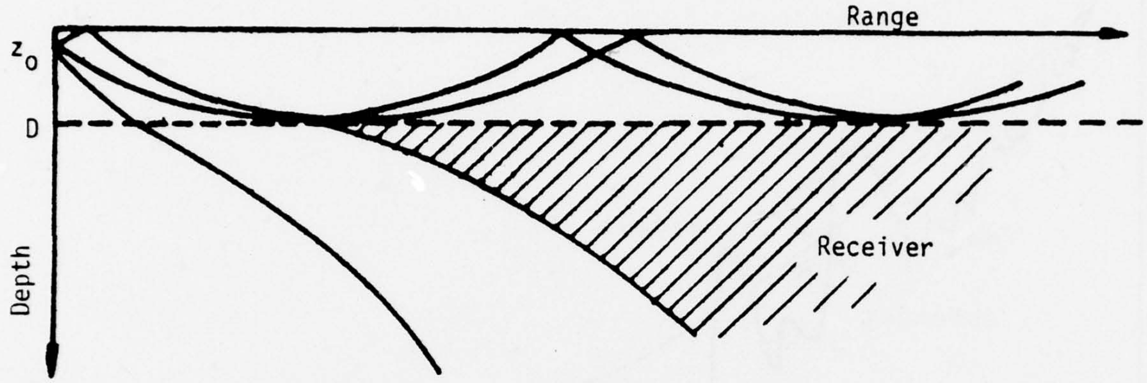
COHERENT summing may cause significant peaks to appear in the transmitted intensity at certain ranges. Such constructive interference would appear, for example, in a convergence zone as shown in the figure. Conversely, at other specific ranges phase differences may cause destructive interference, leading to dips in the intensity curve (and in the TL at those ranges).

INCOHERENCE between the normal modes, on the other hand, may result from the presence of rough boundaries or inhomogeneities within the water layer. Incoherent summing will not produce such detailed structure in the TL curve. The latter will merely fall off with range in a relatively smooth manner.

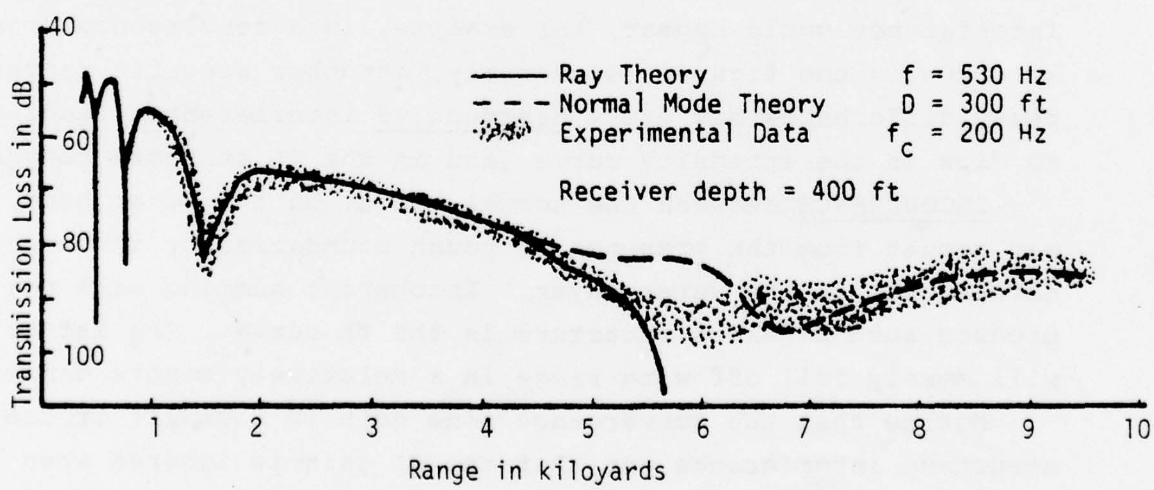
Notice that the convergence zone gain is a result of constructive interference and that the CZ gain is ignored when it is assumed that the pressures for each normal mode combine randomly.

RAYS, NORMAL MODES AND REALITY

Normal Mode Theory is a complicated mathematical formalism which does not lend itself to simple intuitive interpretation. However, it is an exact theory which avoids certain approximations inherent in ray theory. It is useful in describing situations for which ray theory fails. An example of one such situation is illustrated below.



The source is in the mixed layer and the receiver is below the layer. Ray theory predicts a shadow zone at large enough ranges. While there can be some scattering from the ocean surface into the shadow zone, this will be small for low frequencies and low sea states. The surface interference pattern at short ranges shows that the sea was quite smooth (the observed sea state was about 1 or 2).

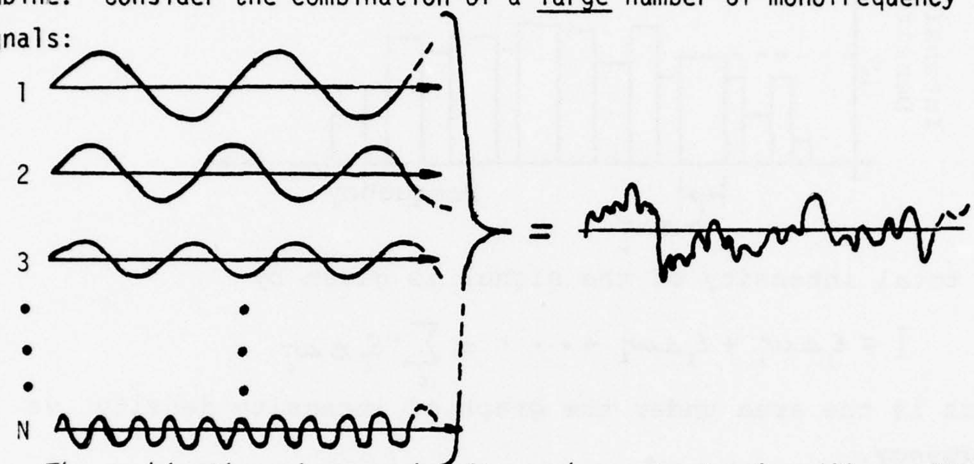


Notice that the experimental data show significant ensonification in the shadow zone. This cannot be accounted for by ray theory, and the scattering of sound from the surface is not strong enough to account for it. Normal Mode Theory does, however, succeed in predicting the ensonification of the shadow zones.

Reference: Pederson and Gordon, Jour. Acoust. Soc. Am. 37, 105 (1965).

COMBINING SIGNALS

Now, let us no longer restrict ourselves to tonals (monofrequency signals). Because real sounds frequently consist of many simultaneous frequencies, we have to find out how signals of different frequencies combine. Consider the combination of a large number of monofrequency signals:



The combination of many signals, each a pure tonal, will result in a total signal which does not appear to have definite amplitude or period. However, the composite signal is still an acoustical wave, and the total acoustical intensity it possesses is the sum of the acoustical intensities of the individual tonals. Thus, if I_T is the total intensity of the composite signal, we have

$$I_T = I_1 + I_2 + \cdots\cdots = \sum_{i=1}^{N} I_i$$

where I_1 is the intensity of signal 1, I_2 that of signal 2, and so on.

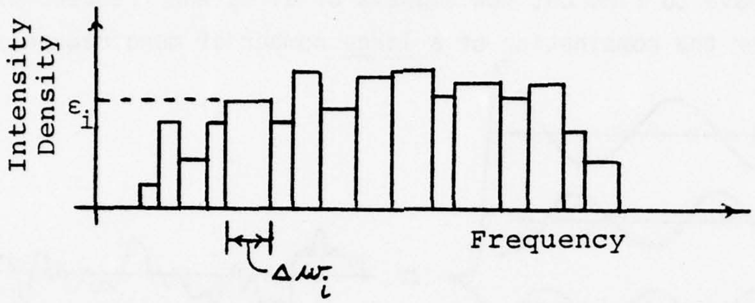
For the complicated signals encountered in real life, which have large variations from moment to moment in their amplitude and shape, the frequencies of the individual components which combine to form them are extremely closely spaced. Indeed, it is often more convenient to talk about the intensity contained within a specific frequency band which is infinitesimally wide. If ΔI is the total intensity of all the components lying within the incremental bandwidth Δw, then the average intensity density over the incremental bandwidth Δw is

$$\varepsilon = \frac{\Delta I}{\Delta w}.$$

Notice that the intensity density has the units of intensity/Hz. Complicated signals, therefore, can be described in terms of the intensity density in each incremental interval of frequency over the entire range of frequencies present.

INTENSITY SPECTRUM LEVEL AND PRESSURE SPECTRUM LEVEL

An example of the analysis of a complex signal is suggested in the following figure:



The total intensity of the signal is given by

$$I = \varepsilon_1 \Delta\omega_1 + \varepsilon_2 \Delta\omega_2 + \cdots = \sum_i \varepsilon_i \Delta\omega_i$$

which is the area under the graph of intensity density vs frequency.

If all of the Δw_i are chosen to be 1 Hz wide, then the graph gives ε, the intensity to be found in each 1 Hz interval, and the curve is called the <u>intensity spectrum</u> of the signal.

Signals are conventionally described in terms of <u>levels</u>. The <u>intensity spectrum level</u> is

$$ISL \text{ re } I_{ref} = 10 \log \frac{\varepsilon \cdot 1\,Hz}{I_{ref}}$$

while the <u>pressure spectrum level</u> is

$$PSL \text{ re } P_{ref} = 20 \log \frac{\sigma}{P_{ref}} \,.$$

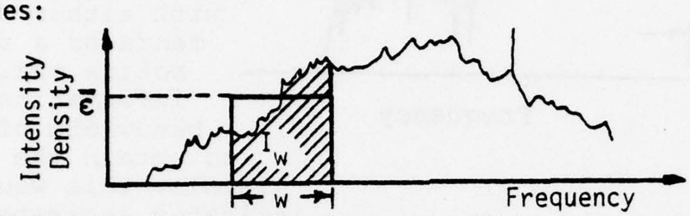
The quantity σ is the effective pressure amplitude that generates the associated intensity found <u>in the 1 Hz bandwidth</u>.

If $I_{ref} = \dfrac{P_{ref}^2}{\rho c}$, then	$ISL = PSL$

AVERAGED INTENSITY SPECTRUM AND AVERAGED PSL

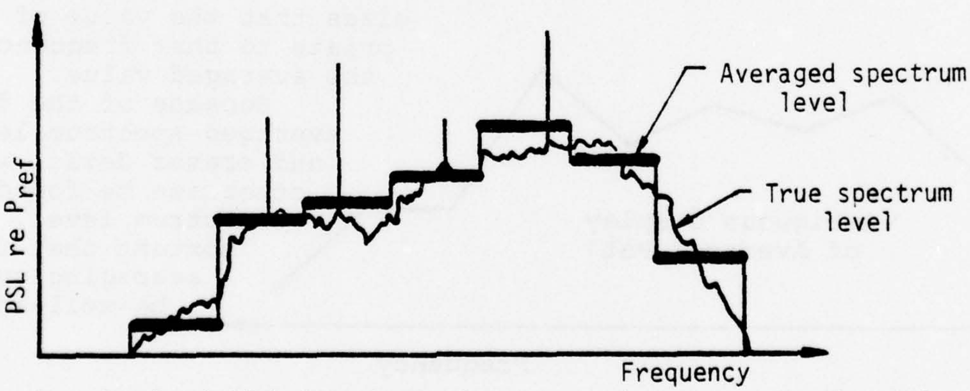
Very often, the intensity spectrum level or the pressure spectrum level is not measured with a 1 Hz filter, but is calculated from measurements made with a filter of larger bandwidth.

The overall intensity in any arbitrary bandwidth w is the area under the intensity spectrum vs frequency curve over the appropriate range of frequencies:



This gives the total intensity I_w found within this bandwidth, and does not say how ε is distributed within w. We can, therefore, define the <u>averaged intensity spectrum</u> $\bar{\varepsilon}$ as $\bar{\varepsilon} = I_w/w$. A plot of $\bar{\varepsilon}$ vs frequency, while lacking the detail of the true curve, is sufficient to calculate the total energy present over bandwidths comparable to or larger than the bandwidths selected to obtain $\bar{\varepsilon}$.

Analogously, what is often presented as a pressure spectrum level is in reality a smoothed curve, the result of plotting data obtained with filters having bandwidths appreciably larger than 1 Hz:

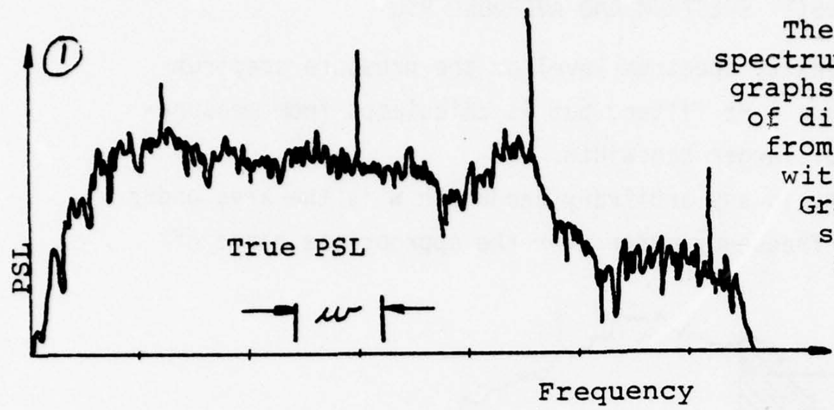


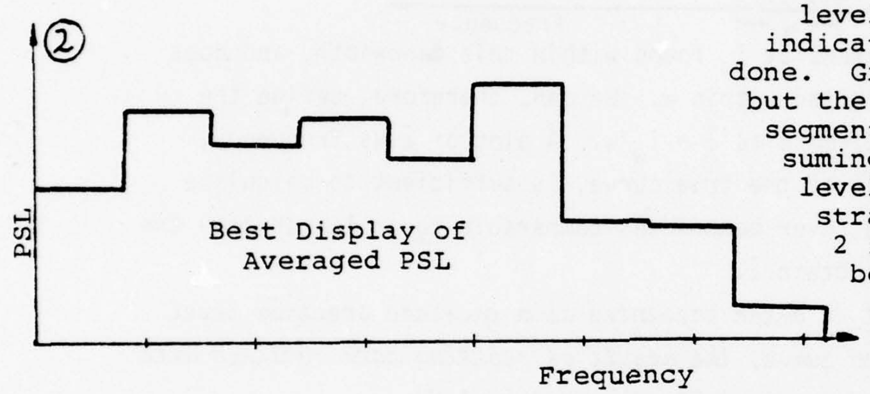
Use of too broad a bandwidth in calculating averaged spectrum levels will wash out details. The spikes in the true spectrum level in the above example are lost to view because the bandwidth over which the averaged levels are taken is so wide that the intensity of the spike is small compared to the intensity of the background in each bandwidth.

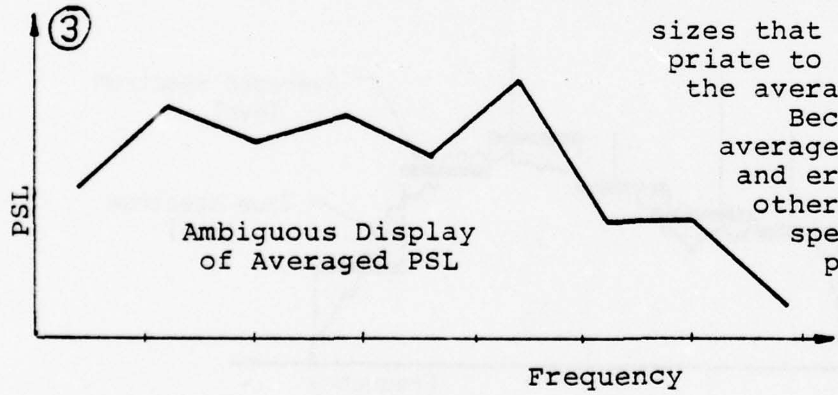
DISPLAYING PRESSURE SPECTRUM LEVELS

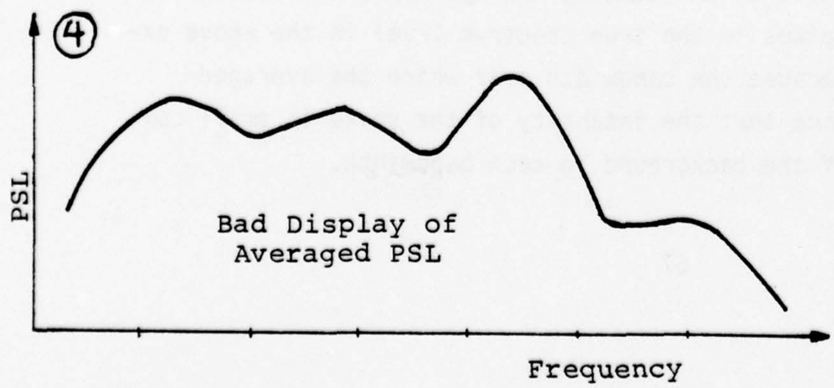
The first graph presents the true spectrum level. The remaining three graphs represent three different ways of displaying the PSL calculated from a single set of data obtained with a filter of bandwidth w. Graph 2 presents the averaged spectrum level as a bar graph. Graphs 3 and 4 connect the data with either straight-line segments or a smooth curve. Notice that Graph 4 gives no information relating to the bandwidth of the filter used to obtain the averaged spectrum levels; this would have to be indicated separately, and is seldom done. Graph 3 is somewhat better, but the use of the straight-line segments seduces the eye into assuming that the true spectrum level closely follows the straight-line segments. Graph 2 is the least subject to being misinterpreted: The width of the individual bars gives the bandwidth of the filter quite visually, and each bar by its flat top strongly emphasizes that the value of the PSL appropriate to that frequency interval is the averaged value.

Because of the fact that the averaged spectrum level smooths out and erases detail which would otherwise be found in the true spectrum level, it is very important that the extent of averaging over frequency be well-indicated.









SPECTRUM LEVEL AND BAND LEVEL

Recall that the total intensity I in a bandwidth w is the area under the intensity density vs frequency curve. The analogous quantity in terms of levels is the <u>band level</u> BL,

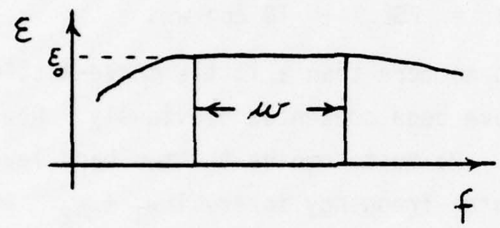
$$BL = 10 \log \frac{I}{I_{ref}}$$

NOTICE THAT THE BAND LEVEL IS <u>NOT</u> THE AREA UNDER THE ISL vs FREQUENCY CURVE.

This is because the *log* of a product is the sum of the *logs* of the individual factors:

$$\log xy = \log x + \log y$$

Consider a plot of intensity density ε vs frequency as shown:



It is clear that the total intensity I in the bandwidth w is given by $I = \varepsilon_o w$. If we express this in *log* form, we obtain

$$10 \log \frac{I}{I_{ref}} = 10 \log \frac{\varepsilon_o \cdot 1\,Hz}{I_{ref}} + 10 \log \frac{w}{1\,Hz} .$$

The left hand term is an intensity level for the intensity within the band of frequencies w. We now call this level a <u>band level</u>, BL. The first term on the right is the ISL previously defined; it is generally identical with the PSL. Thus

$$\boxed{BL = PSL + 10 \log w}$$

where the 1 Hz in the denominator of the last term is understood.

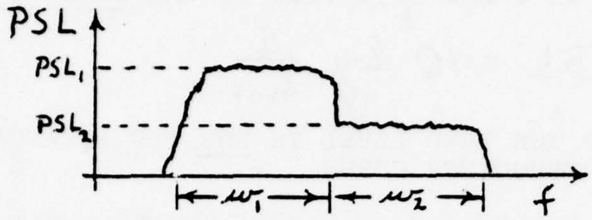
In general this expression is exact <u>only</u> if the spectrum level is constant over the bandwidth w. <u>However</u>, it is reasonably accurate if the true spectrum level does not fluctuate by more than a few dB over the range of w.

Notice that this formula can be turned around: If a band level BL has been experimentally obtained by measuring the acoustical energy over some bandwidth w, then the averaged spectrum level over this bandwidth can be found from

$$PSL \text{ (averaged)} = BL - 10 \log w$$

A SAMPLE CALCULATION

As a simple example, let us obtain the overall band level for the PSL curve presented below:



From the steplike nature of the curve, we can specify two bandwidths w_1 and w_2 as shown, with the associated averaged spectrum levels. For each of these, we can calculate the appropriate band level:

$$BL(w_1) = PSL_1 + 10 \log w_1$$
$$BL(w_2) = PSL_2 + 10 \log w_2 .$$

So far, we have had to do no more than a little curve-fitting and application of formulas which have been presented previously. Now, however, a new problem has appeared. We must combine the two band levels into a single band level for the total frequency interval $w_1 + w_2$. The combined BL is <u>not</u> the sum of the individual ones, because we are in dB: *Intensities add but logs of intensities do not.* What must be done is to determine the intensity in $BL(w_1)$ and the intensity in $BL(w_2)$, add these together, and express the result as a band level.

Let us illustrate this with an example. We shall first do the example in the method indicated above, and then present a simple nomogram which avoids all the work. If

$$PSL_1 = 140 \text{ dB re } 1\mu\text{Pa} \qquad w_1 = 1000 \text{ Hz}$$
$$PSL_2 = 135 \text{ dB re } 1\mu\text{Pa} \qquad w_2 = 2000 \text{ Hz}$$

then

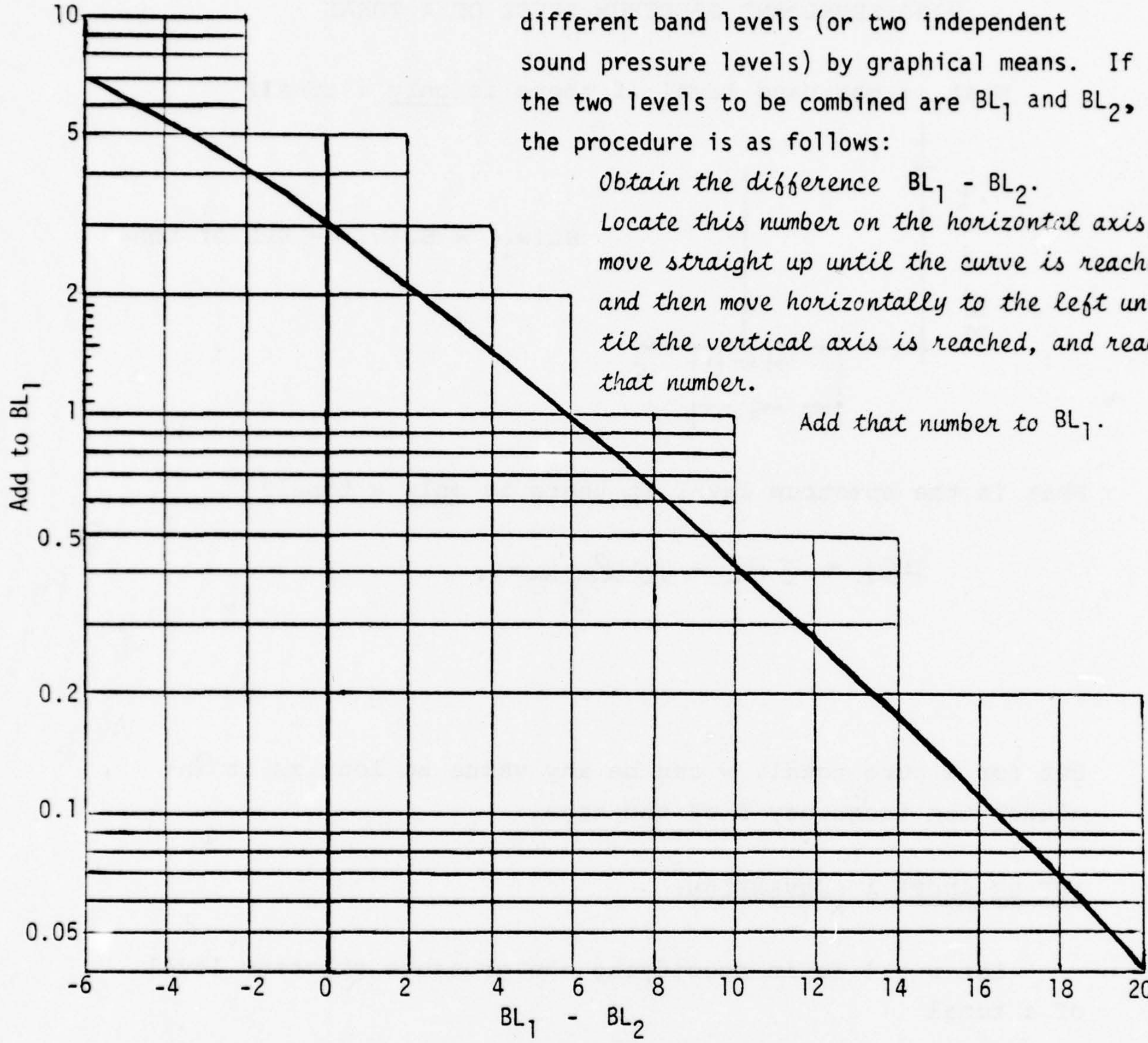
$$BL(w_1) = 140 + 10 \log 1000 = 170 \text{ dB re } 1\mu\text{Pa}$$
$$BL(w_2) = 135 + 10 \log 2000 = 168 \text{ dB re } 1\mu\text{Pa}$$
$$I(w_1)/I_{ref} = \text{antilog } \frac{170}{10} = 10^{17}$$
$$I(w_2)/I_{ref} = \text{antilog } \frac{168}{10} = 10^{16.8}$$
$$I/I_{ref} = \frac{I(w_1) + I(w_2)}{I_{ref}} = 10^{17} + 10^{16.8}$$
$$BL(w_1 + w_2) = 10 \log (10^{17} + 10^{16.8}) = \underline{172.1 \text{ dB re } 1\mu\text{Pa}} .$$

NOMOGRAM FOR COMBINING LEVELS

This nomogram allows the combination of two different band levels (or two independent sound pressure levels) by graphical means. If the two levels to be combined are BL_1 and BL_2, the procedure is as follows:

Obtain the difference $BL_1 - BL_2$. *Locate this number on the horizontal axis, move straight up until the curve is reached, and then move horizontally to the left until the vertical axis is reached, and read that number.*

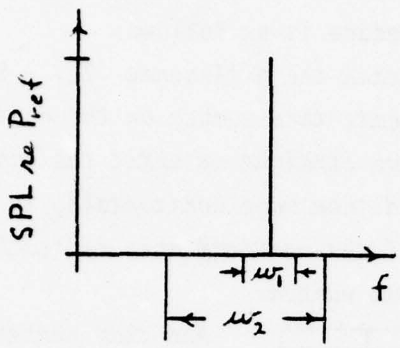
Add that number to BL_1.



As an example of the application of this nomogram, recall the problem worked out on the previous page. The two band levels which were calculated there were BL_1 = 170 dB re 1μPa and BL_2 = 168 dB re 1μPa. The difference is +2 dB; finding this value, projecting upward, and reading to the left gives 2.1 dB, so that the combination of these two levels yields a total band level of 170 + 2.1 = 172.1 dB re 1μPa. Since the usual uncertainty in levels exceeds a few tenths of a dB, it is appropriate to round this combined level to 172 dB re 1μPa.

BAND LEVEL AND SPECTRUM LEVEL OF A TONAL

What is the band level if there is only a tonal?



$BL(w_1) = BL(w_2) =$ SPL of tonal

What is the spectrum level if there is only a tonal?

$$PSL = SPL - 10 \log w .$$

But for a pure tonal, w can be any value so long as it includes the frequency f of the tonal.

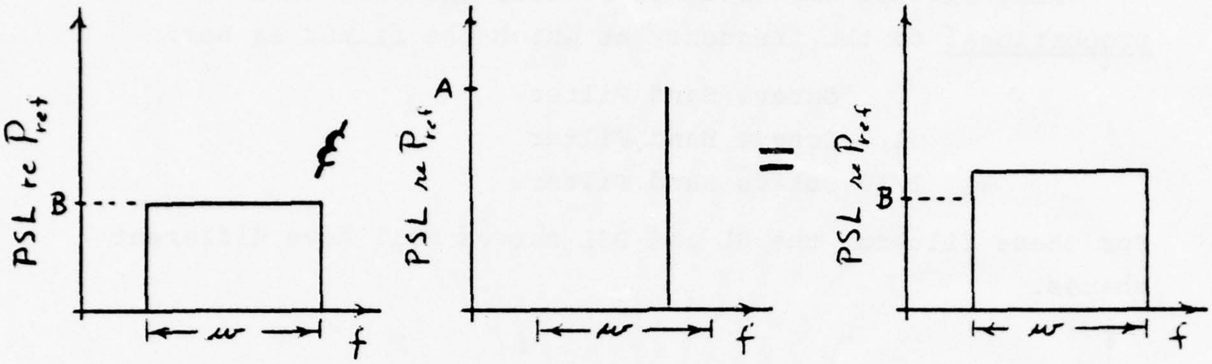
LET US ADOPT A CONVENTION:

Let w = 1 Hz in specifying the pressure spectrum level of a tonal

$$\boxed{PSL\,(tonal) \equiv SPL\,(tonal)}$$

This allows us to plot the levels of tonals on spectrum level graphs and calculate overall band levels.

TONALS COMBINED WITH BACKGROUND



When there are strong tonals present, the band level cannot be expressed as B + 10 ℓog w, because of the additional intensity contributed by the tonals. We must calculate the intensity from the background spectrum (without the tonals) and add this to the intensity of the tonals which are also present. Equivalently, we find the bandwidth of each and combine them:

$$BL_B = B + 10 \log w$$

$$BL_A = A + 10 \log 1 = A$$

The total band level is found by combining BL_A and BL_B with the help of the nomogram.

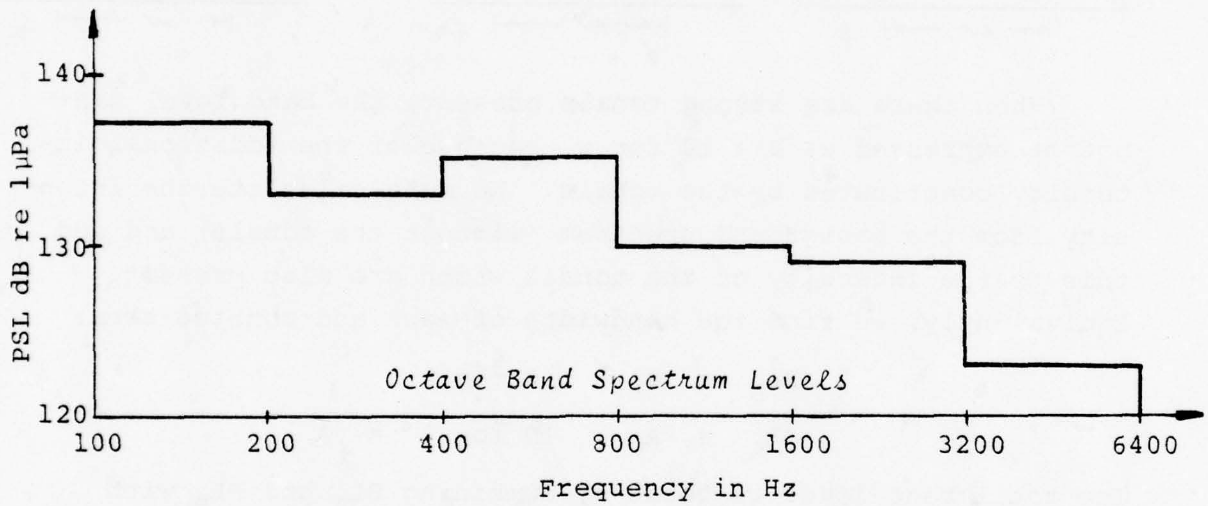
Notice that if the tonal is relatively strong and the bandwidth narrow enough, the combined level will be due essentially to the tonal. As the bandwidth w increases, however, the influence of the (10 ℓog w) term will become stronger, causing BL_B to increase. Eventually the contribution from the background will get so large that BL_B will dominate and the influence of BL_A will be lost.

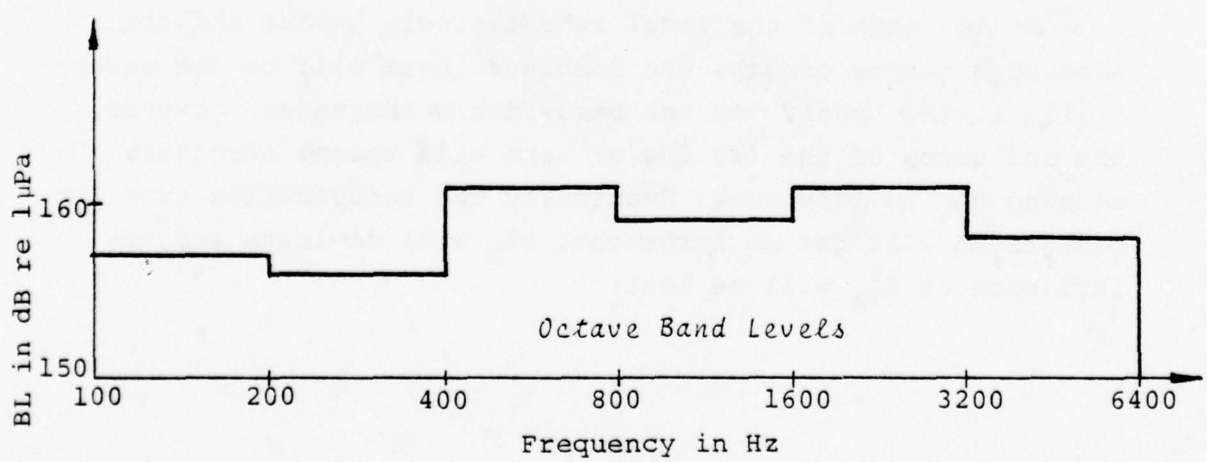
PROPORTIONAL FILTERS

Many filters are designed so that the bandwidth is <u>proportional</u> to the frequency at which the filter is set:

Octave Band Filter
1/3 Octave Band Filter
1/10 Octave Band Filter.

For these filters, the BL and PSL curves will have different shapes.



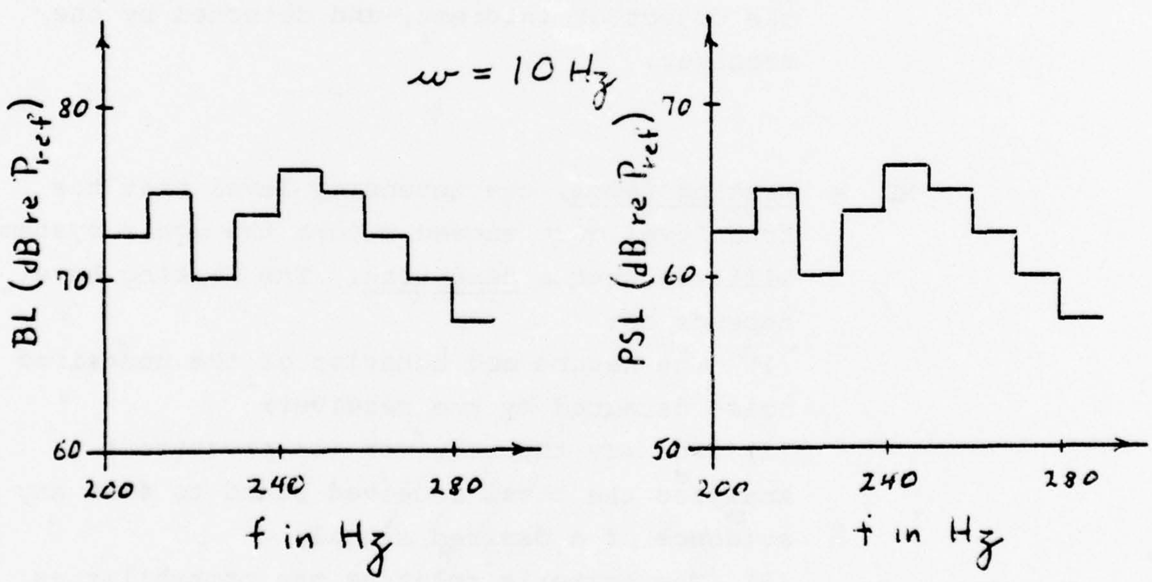


CONSTANT BANDWIDTH FILTERS

Certain other filters are designed to have <u>constant</u> bandwidth. For these filters,

$$PSL = BL - 10 \log w = BL - \text{constant}$$

So that if the band levels are recorded and plotted, the PSL curve will have the <u>same shape</u>, but different absolute values:



THE SONAR EQUATION

The basic problem to be solved in the detection of underwater targets is to fulfill the requirements of the SONAR EQUATION

$$\boxed{EL \geq ML}$$

where EL = Echo Level, the level of the desired signal detected by the receiver. In the case of passive sonar this is the sound which has been radiated by the object of interest and detected. For active sonar it is the tone burst which has been generated by the transmitter, reflected by the object of interest, and detected by the receiver.

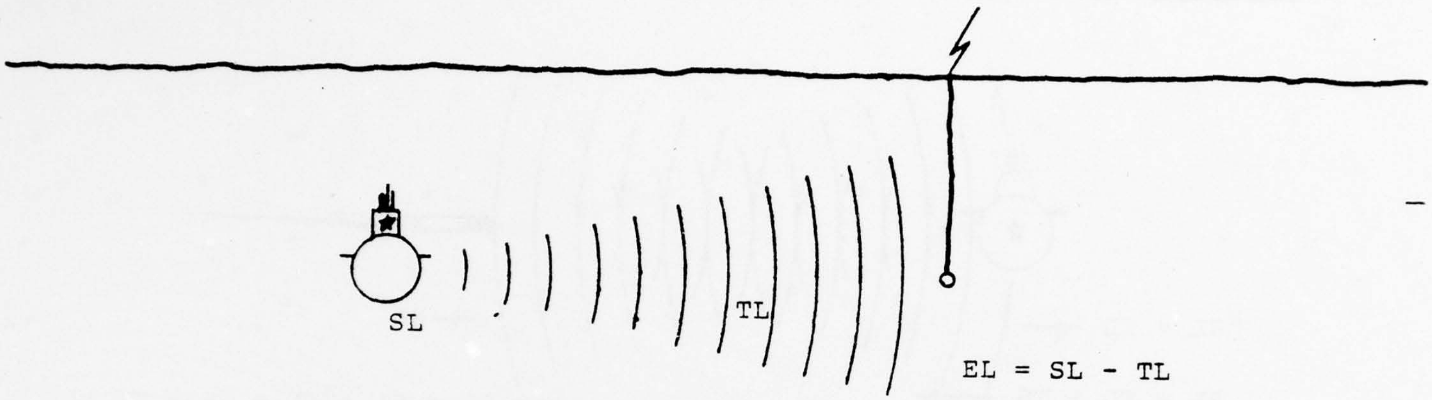
ML = Masking Level, the intensity level that the Echo Level must exceed before the sonar system will register a detection. The Masking Level depends on:

(1) The nature and behavior of the undesired noise detected by the receiver;

(2) The way the receiver (or processor) analyzes the total received sound to find any evidence of a desired signal;

(3) The criteria relating the probabilities of guessing rightly or wrongly that an object of interest is indeed present.

PASSIVE SONAR

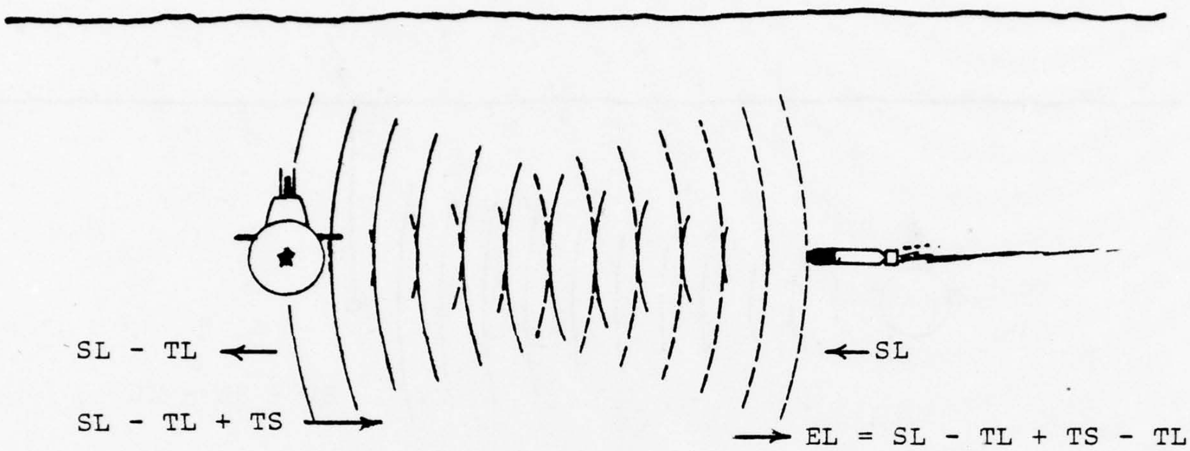


$$\boxed{SL - TL \geq ML}$$

where SL = Source Level of the target

TL = One-way Transmission Loss

ACTIVE SONAR



$$\boxed{SL - 2TL + TS \geq ML}$$

where

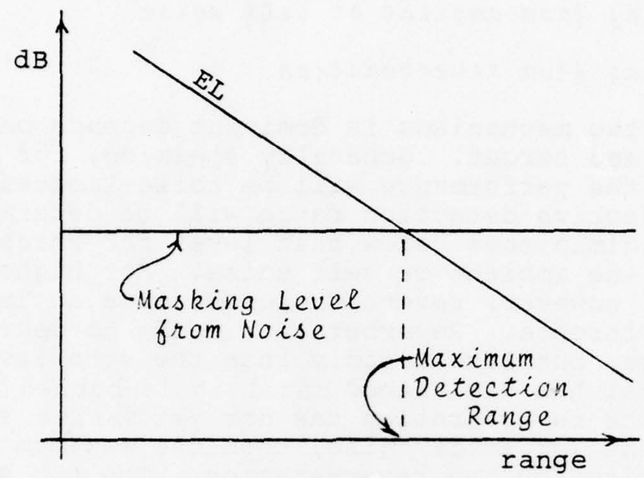
SL	=	Source Level of transmitter
2TL	=	two-way Transmission Loss
TS	=	Target Strength, the measure in dB of the ability of the target to reflect sound toward the receiver.

The above active SONAR equation is for a monostatic case, wherein the source and receiver are coincident so that the TL from source to target is the same as the TL from target to receiver. Our discussion will usually be restricted to this case. Extension to the bistatic situation in which source and receiver are at different locations is trivial: If the transmission loss from source to target is TL, and from target to receiver TL', then all that is required is to replace 2TL with TL + TL', so that we have

$$\boxed{SL - (TL + TL') + TS \geq ML}$$

MASKING LEVEL (PASSIVE)

In the case of passive SONAR, the only masking mechanism is ambient- or self-noise:



As range between the source of sound and the passive sonar increases, the echo level will tend to diminish (although it may increase if there is convergence zone capability and the range between source and receiver is the convergence zone range). The received noise, however, arises from the ambient noise of the ocean or the noise generated by the receiving platform so that it is, on the average, independent of range. When the echo level has fallen below the noise masking level, then the source cannot be detected and the maximum detection range has been reached.

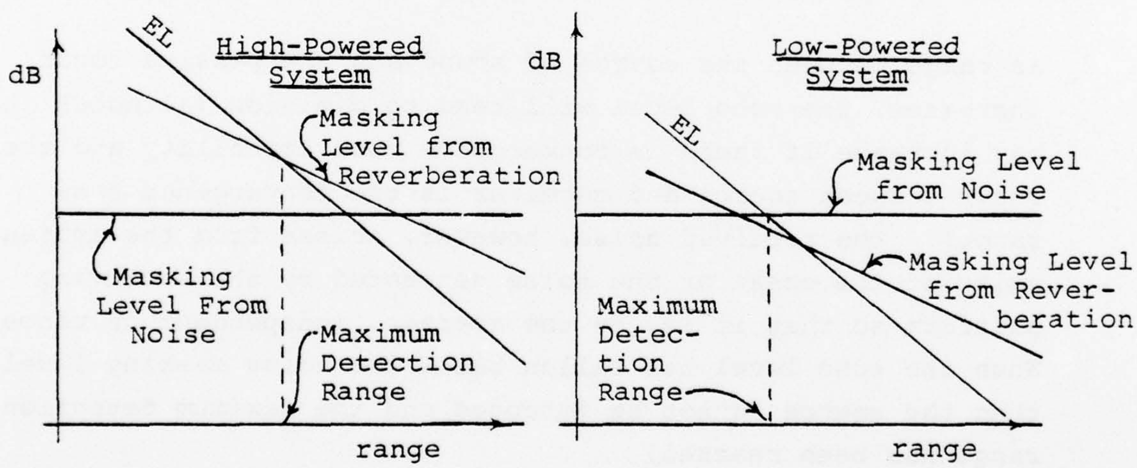
MASKING LEVEL (ACTIVE)

The undesired noise competing with the desired signal can arise from two possible mechanisms in the case of active SONAR:

Masking from ambient or self noise

Masking from reverberation.

Which of these two mechanisms is dominant depends on the range between source and target. Generally speaking, for low-powered active systems the performance will be noise-limited, in that the maximum effective detection range will be determined when the echo level diminishes below that level for which it can be extracted from the ambient or self noise. For higher-powered active systems, however, reverberation becomes an important source of interference. Reverberation tends to decrease with increasing range, but less rapidly than the echo level. Thus, if the echo level has diminished until it is buried in the reverberation, but the reverberation has not yet fallen sufficiently below the ambient (or self) noise, then the maximum detection range is determined by the reverberation. The two situations are suggested in the figures:



The choice is a little more subtle than suggested by the above simple curves. If there is a Doppler-induced difference in frequency between the echo received from the target and the reverberation, and if it is possible in the receiver to discriminate between these two frequencies, then the masking of the echo can arise from the ambient noise at that frequency. If the target is stationary, however, the masking is from the reverberation, since echo and reverberation then have the same frequency.

DETECTED NOISE LEVEL

The masking level can also be written

$$ML = DNL + DT$$

where DNL = Detected Noise Level, the Intensity Level in dB of the undesired signal as presented to the decision-making system

and DT = Detection Threshold, the number of dB by which the Echo Level must exceed the Detected Noise Level to allow a detection to be registered with a specific degree of confidence.

The Sonar Equation then becomes

$$EL \geq DNL + DT$$

or

$SL - TL \geq DNL + DT$	passive
$SL - 2TL + TS \geq DNL + DT$	active

FIGURE OF MERIT
FOM

If the SONAR equations are rearranged into the forms isolating the transmission losses,

$$2TL \leq SL + TS - ML \qquad \text{ACTIVE,}$$

or $$TL \leq SL - ML \qquad \text{PASSIVE,}$$

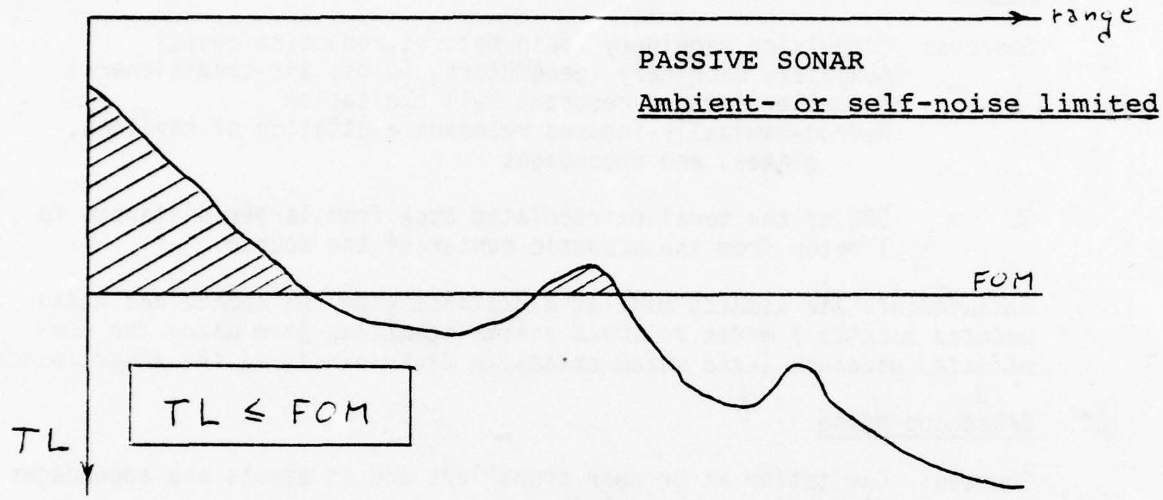
then the right-hand sides are termed the Figures of Merit, or

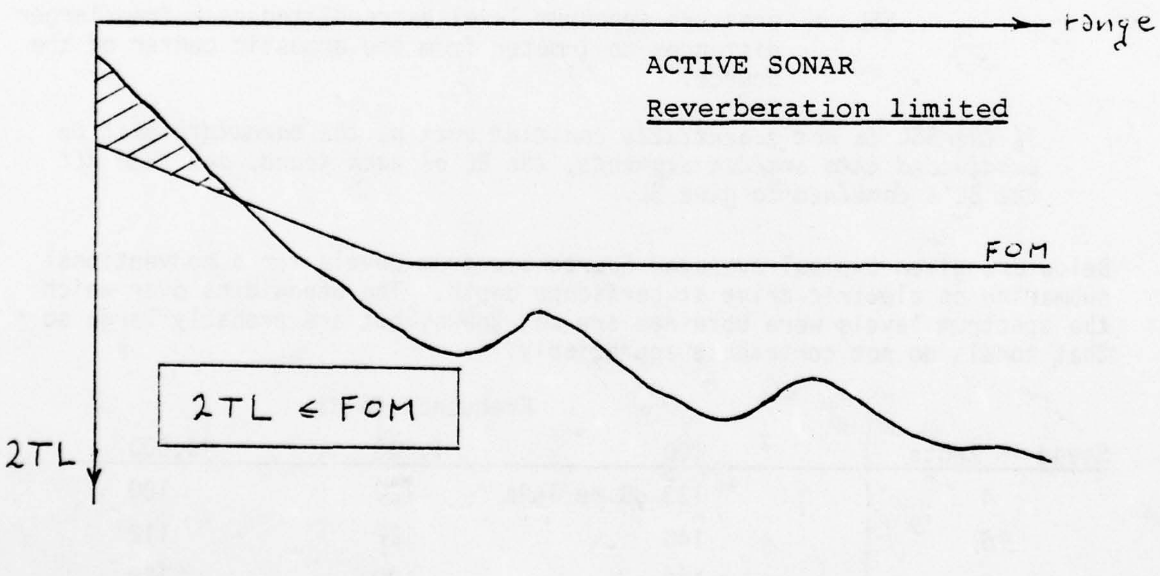
$$\boxed{FOM = \begin{cases} SL + TS - ML & \text{ACTIVE} \\ SL - ML & \text{PASSIVE.} \end{cases}}$$

For ambient- or self-noise limited performance, FOM is a number independent of range which can be calculated for the particular SONAR system being considered. If TL for passive sonar, or 2TL for active, exceeds the FOM, then detection is not possible to within the desired probability of success.

For active SONAR under reverberation-limited performance, FOM is a function of range, but it still provides the same numerical criterion that 2TL must satisfy in order that detection be achieved.

TWO EXAMPLES





 ranges for which detections are possible for conditions equal to or better than those specified in ML.

SOURCE LEVEL (PASSIVE)

SL - TL ≥ ML

Noise sources radiate sounds of two distinct types:

I. Tonals (monofrequency components)

Sources: Propulsion machinery (main motors, reduction gears)
Auxiliary machinery (generators, pumps, air-conditioners)
Propeller-induced resonant hull excitation
Hydrodynamically-induced resonant excitation of cavities, plates, and appendages

SL = SPL of the tonal extrapolated back from larger distances to 1 meter from the acoustic center of the source.

Measurements are usually made at a distance from the source and extrapolated back to 1 meter to avoid errors resulting from using the complicated pressure field which exists in the vicinity of the large source.

II. Broadband Noise

Sources: Cavitation at or near propellers and at struts and appendages
Flow noise

SL = SSL + 10 ℓog w

where w is the bandwidth of the system and

SSL = pressure spectrum level extrapolated back from larger distances to 1 meter from the acoustic center of the source.

If the SSL *is not essentially constant over* w, *the bandwidth must be subdivided into smaller segments, the* BL *of each found, and then all the* BL'*s combined to give* SL.

Below are given typical averaged Source Spectrum Levels for a conventional submarine on electric drive at periscope depth. The bandwidths over which the spectrum levels were obtained are not known, but are probably large so that tonals do not contribute appreciably.

	Frequency in Hz		
Speed in knots	100	1,000	10,000
4	133 dB re 1μPa	120	100
6	140	127	112
8	143	133	119
10	147	135	122

Adapted from Urick, Principles of Underwater Sound for Engineers.

SOURCE LEVEL (ACTIVE)

$$SL - 2TL + TS \geq ML$$

SL = Sound Pressure Level 1 m from the acoustic center of the transmitting transducer, as found by extrapolating the pressure amplitude back from larger distances.

$$\boxed{SL \text{ re } 1\,\mu Pa = 10 \log(\text{Power}) + DI + 171 \text{ dB}}$$

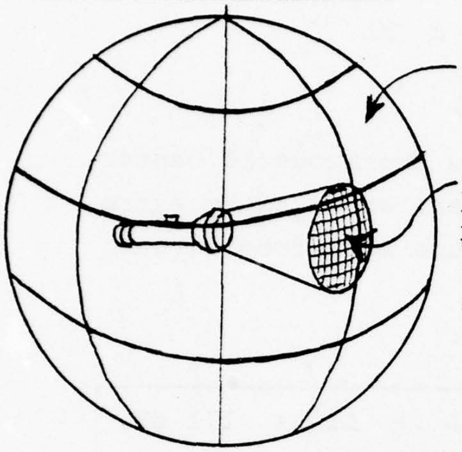
where 171 dB = A constant which takes care of the units.

Power = Acoustic power output of the transducer in Watts.

DI = *Directivity Index*. This is a measure of how well the source is able to channel the acoustical power into some specific direction (rather than radiating it out in all directions).

DIRECTIVITY INDEX (DI)

The Physical Meaning:



Surface area of the sphere = $4\pi r^2$.

Surface area of the illuminated patch = Ωr^2.

The fractional area of the sphere which is illuminated is $\Omega/4\pi$. The inverse of this is the directivity D, which, for this simple example, is given by

$$D = \frac{4\pi}{\Omega} .$$

The Definition:

The pressure field generated by an array acting like a source, or the sensitivity of an array acting like a receiver, can be written in the form

$$\boxed{P(r,\theta,\phi) = P_{ax}(r)\, H(\theta,\phi)}$$

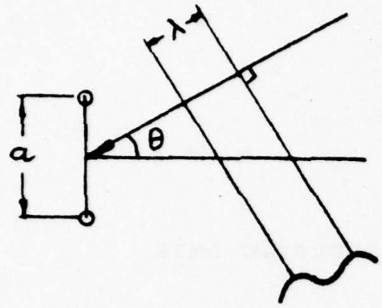
where P_{ax} is the pressure on the acoustic axis of the array, and $H(\theta,\phi)$ is the directional factor that corrects for points off the axis. Since the acoustic axis is defined to be the direction(s) for which the generated pressure field, or the sensitivity, is largest, $H(\theta,\phi)$ is normalized to have a maximum value of unity on the acoustic axis, and to be no greater than unity for any other directions. The directivity is defined as

$$\boxed{D = \frac{4\pi}{\int H^2(\theta,\phi)\, d\Omega}} .$$

The directivity index is the expression of the directivity D in dB terms,

$$\boxed{DI = 10 \log D} .$$

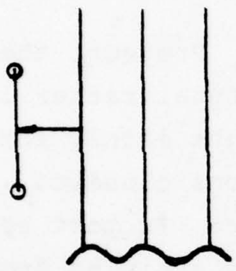
EXAMPLE: AN ARRAY OF TWO RECEIVERS



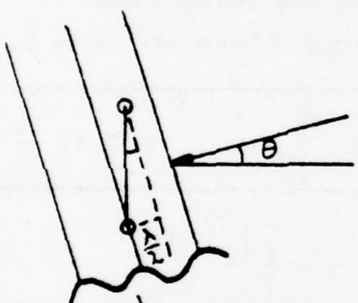
a = Separation of receivers

λ = Wavelength of incoming wave

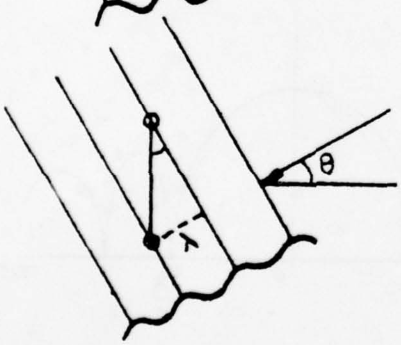
θ = Angle between direction of incoming wave and the perpendicular bisector of the line joining the receivers. (This is equal to the angle between the wave front and the line joining the receivers.)



$\theta = 0$ Both receivers experience the same pressure at the same time, combined output has maximum amplitude.

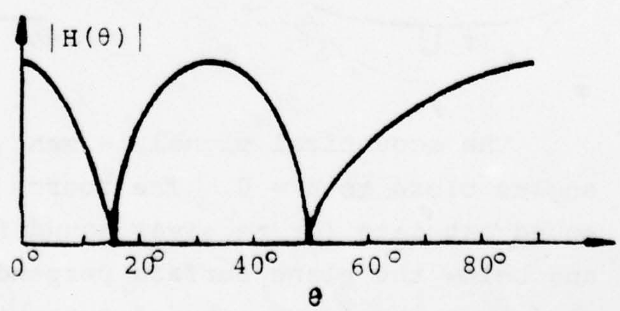


$\sin\theta = \frac{\lambda}{2a}$ Receivers experience pressures 180° out of phase, combined output is zero, and there is a null.



$\sin\theta = \frac{\lambda}{a}$ Receivers again receive pressures in phase and output is again maximum.

If λ = a/2, the array will have a directional factor H(θ) as graphed

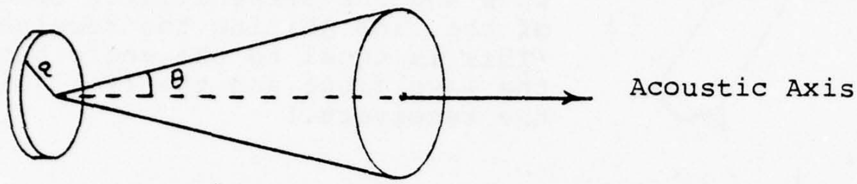


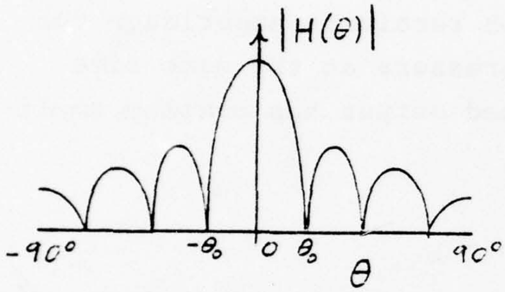
NULLS for the 2-element array when $|\sin\theta| = (n-\frac{1}{2})(\lambda/a)$ *and* $n = 1, 2, \ldots$

$\lambda/a = 0.5$

PISTONS AND LINES

Piston Source or Receiver



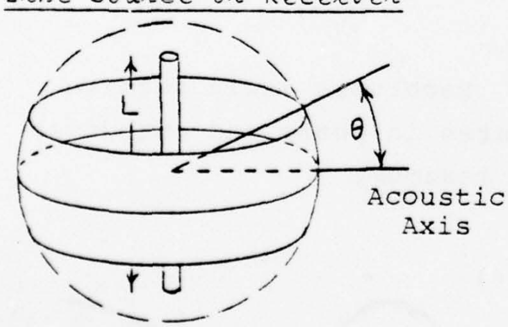


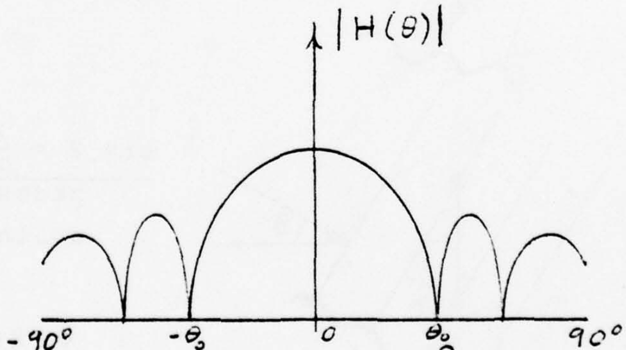
As a source: Projects the acoustical signal rather like a flashlight mainly into directions close to $\theta = 0$.

As a receiver: Is most sensitive to sound arriving from directions close to $\theta = 0$.

$DI \doteq 10 \log \left(\frac{2\pi a}{\lambda}\right)^2$	if $a > \lambda$	$\sin \theta_o = 0.61 \frac{\lambda}{a}$

Line Source or Receiver



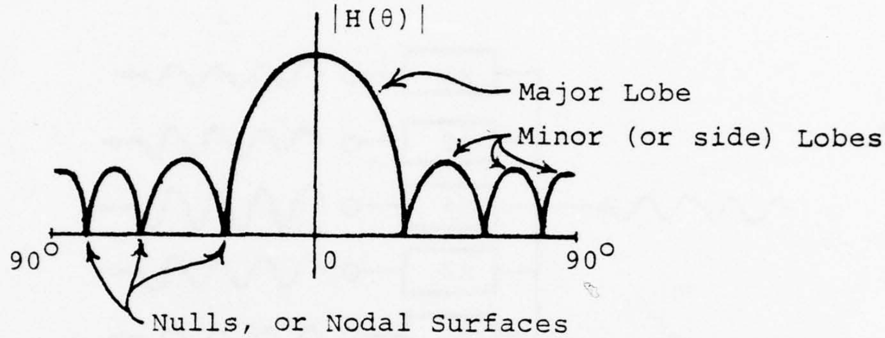


The acoustical signal is sent into (or received from) angles close to $\theta = 0$. The source (or receiver) therefore sends sound out into (or receives sound from) a narrow wedge above and below the plane surface perpendicular to the axis of line source or receiver, in an "equatorial belt".

$DI \doteq 10 \log (2L/\lambda)$	if $L > \lambda$	$\sin \theta_o = \lambda/L$

LOBES AND NULLS

Properties of the Directional Factor

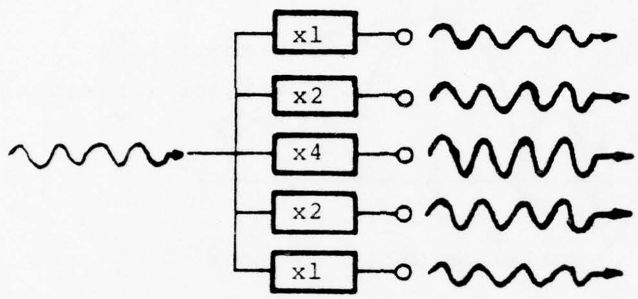


How many side lobes or nodal surfaces there are depends on the size of the source and receiver compared to a wavelength of the sound wave being generated or received. The larger the wavelength is compared to the size, the "fatter" the major lobe and the fewer the side lobes that will be found. The fatter the major lobe is, the greater will be $\int H^2 \, d\Omega$, so that D gets smaller:

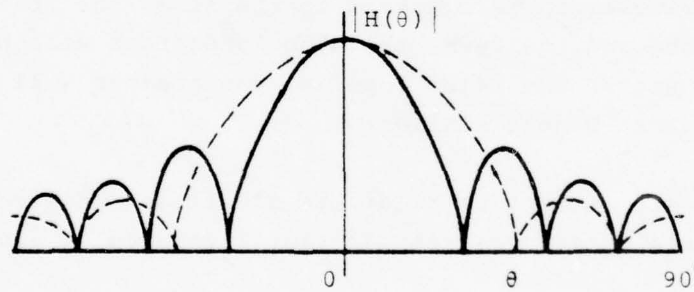
> *Small sources or receivers are less directional than large ones for the same frequency of sound.*

THE SHADED ARRAY

Sources and receivers can be shaded by preferentially amplifying the signals generated or detected by certain of the individual elements of the array:



This changes the relative importance of the side lobes compared to the major lobe:



The response in directions other than that of the major lobe can be reduced or enhanced relative to the major lobe, and the major lobe may be broadened or narrowed, depending on the choice of the amplifications.

THERE IS A TRADE-OFF:

DESIRED EFFECT		PENALTY
Suppress the side lobes	⟷	*Major lobe is broadened*
Reduce angular extent of the major lobe	⟷	*Side lobes will be strengthened*

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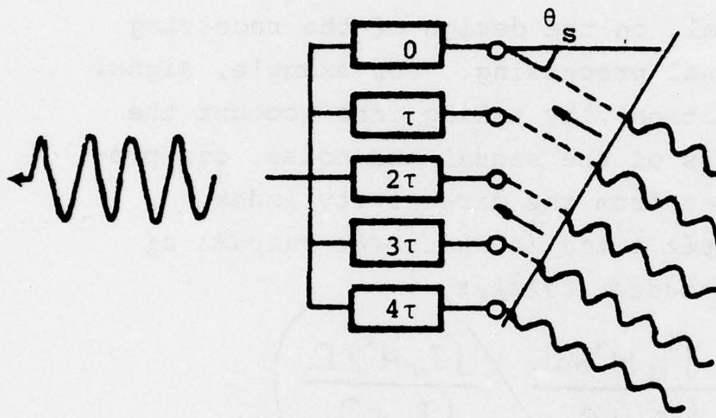
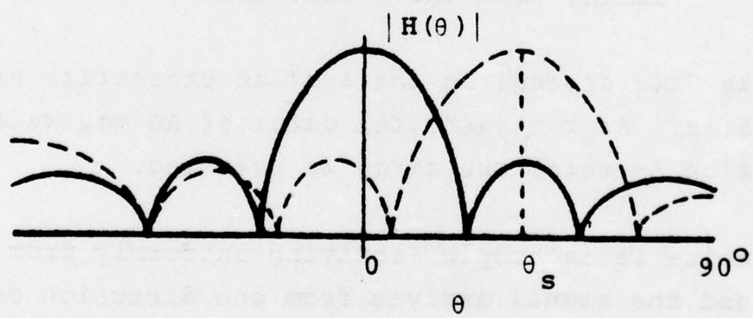
THE STEERED ARRAY

Sources and receivers can be steered by inserting time delays in the signals generated or detected by the individual elements of the array.



This changes the direction of the major lobe, pointing it off at some angle θ_s:



THERE ARE TRADE-OFFS: In order for a major lobe to be steered to large angles, the array must be designed to be less directional than possible when it is unsteered. Otherwise, when it is steered, additional major lobes may appear.

ARRAY GAIN FOR A RECEIVER
AG

Array gain is a measure of the enhancement of the desired signal with respect to the detected noise, brought about by the appropriate design of the receiving array and detection system. It depends on the directional properties of both the noise and the desired signal, on the design of the receiving array, and also on the signal processing. For example, signal processing (such as correlation), by taking into account the different spatial properties of the signal and noise, can provide an array gain different from the directivity index.

For the case of a simple array in which the outputs of the separate receivers are added together,

$$AG = 10 \log \left(\frac{\int I_s H^2 d\Omega}{\int I_s d\Omega} \bigg/ \frac{\int I_N H^2 d\Omega}{\int I_N d\Omega} \right)$$

where $I_N(\Omega)$ = the noise intensity per unit solid angle coming from the Ω direction.

$I_S(\Omega)$ = the signal intensity per unit solid angle coming from the Ω direction.

The Array Gain depends on the spatial properties of both signal and noise. As a result, the value of AG *may well depend on the direction in which the array is pointing.*

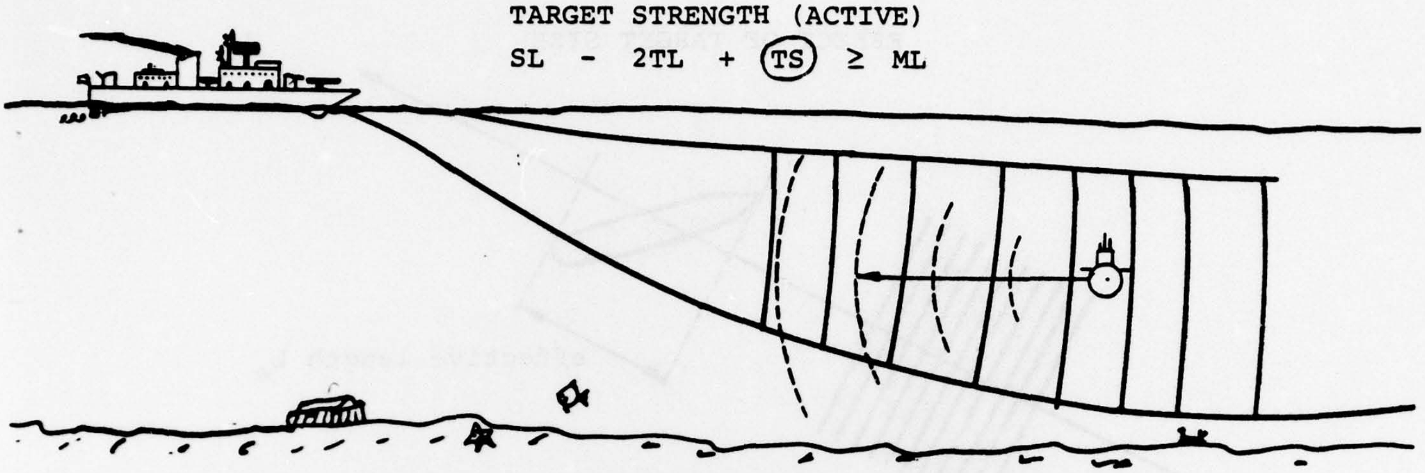
If the noise is isotropic (arriving uniformly from all directions) and the signal arrives from one direction only, then

$$AG = DI$$

for an array in which the outputs of the individual receiving elements are added together.

TARGET STRENGTH (ACTIVE)

SL - 2TL + TS $\geq$ ML



The Target Strength is defined as

$$\boxed{TS = 20 \log\left[P'(r'=1)/P(r)\right]}$$

where P(r) = the amplitude of the pressure of the incident wave that would be measured at the position of the target if the target were absent.

P'(r'=1) = the pressure of the reflected wave extrapolated back to 1 m from the acoustic center of the target.

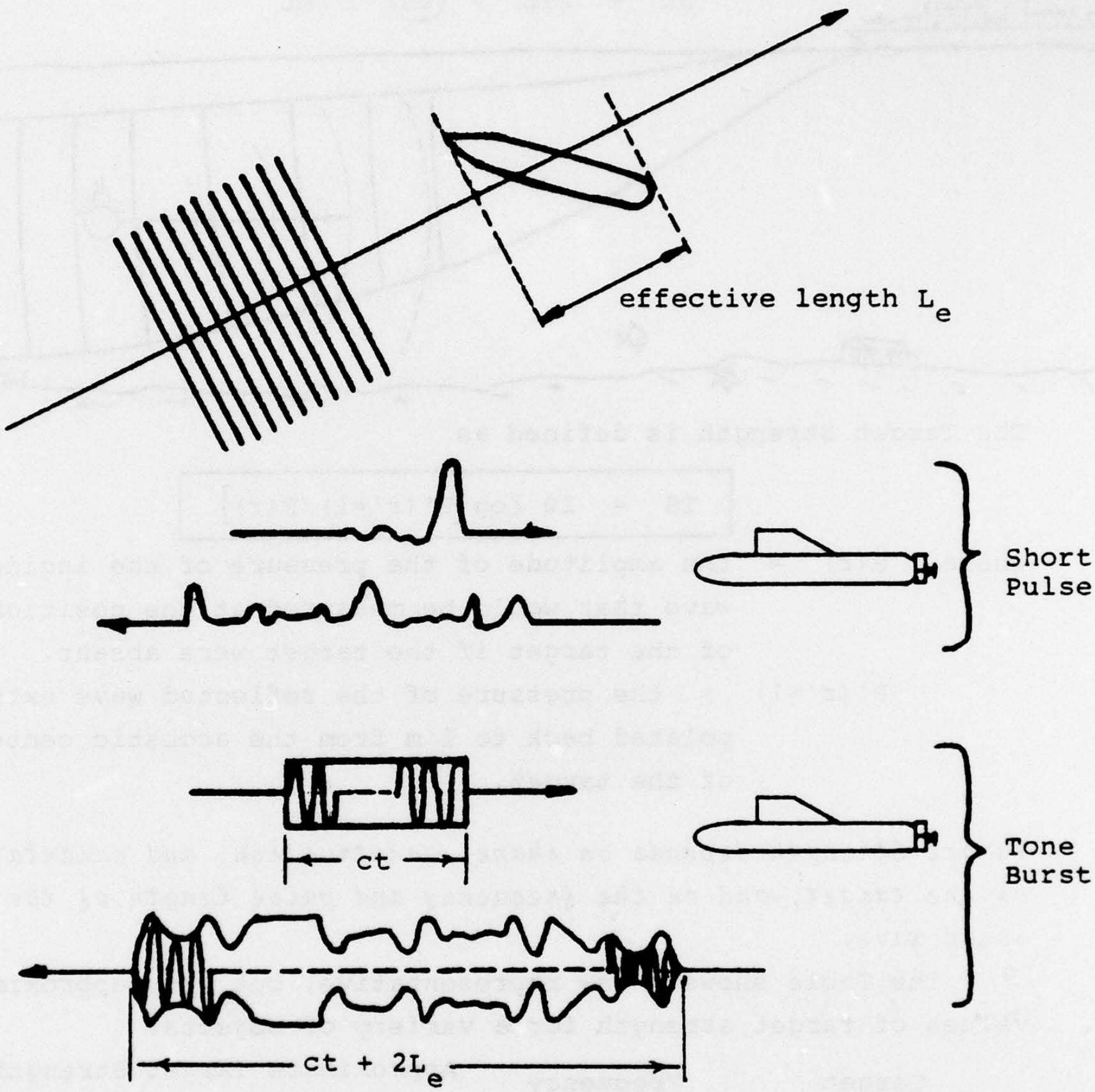
Target Strength depends on shape, construction, and orientation of the target, and on the frequency and pulse length of the sound wave.

The Table shows a few representative, but very approximate, values of target strength for a variety of objects.

Target	Frequency in kHz	Approximate Target Strength (dB) Beam Aspect	Bow-Stern Aspect
Fleet Submarine (approx 100 m long)	10 24	35 25	30 16
S Class Submarine (approx 70 m long)	24	20	13
Torpedo	24		-20
Surface Vessel		10 to 30	
3 ft mine	30-90	-8	

Extracted from: Physics of Sound in the Sea, NDRC
Principles of Underwater Sound for Engineers, Urick
Priciple and Applications of Underwater Sound, NDRC
NAVSHIPS 0967-129-3010

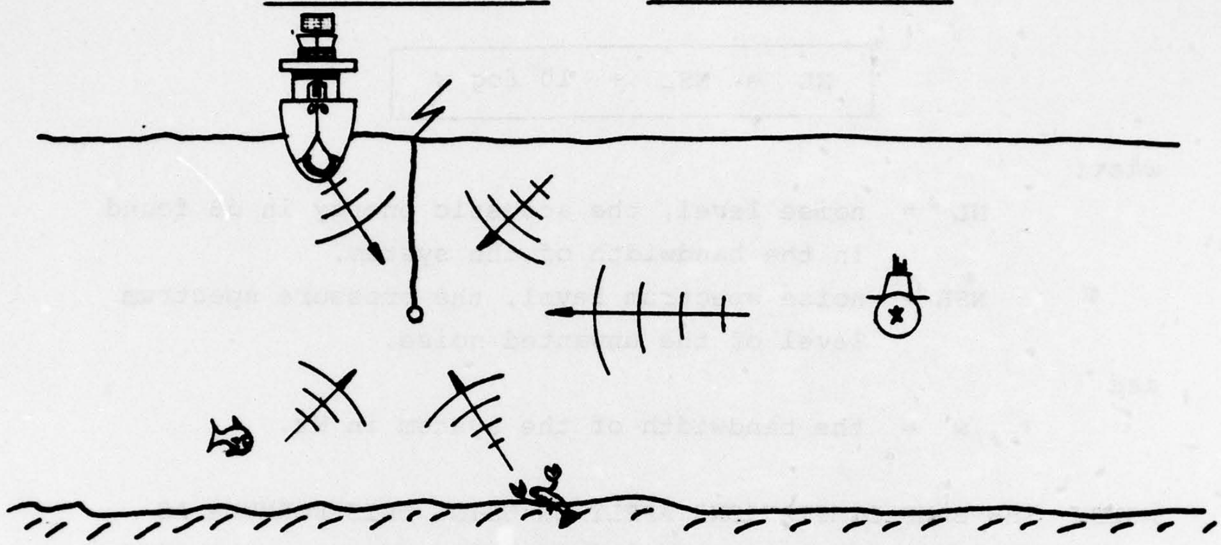
EFFECT OF TARGET SIZE



Because of the effective length L_e of the target, portions of the incident pulse which reflect from the stern (in the above sketches) travel a total distance of $2L_e$ farther than the reflections from the bow. This means that the apparent duration of the pulse is stretched from t to $(t + 2L_e/c)$. For very long pulses, the additional time interval corresponding to the round trip, $2L_e/c$, can be neglected. For short pulses, however, this stretching can be quite important in determining the target strength.

DETECTED NOISE LEVEL

$$SL - TL \geq ML \qquad ML = DNL + DT$$



Noise may come in uniformly from all directions or it may arrive from a preferred direction. If the receiver is sensitive only to noise from some preferred direction, then the noise it will actually detect (DNL) must be the noise level measured by an omnidirectional receiver (NL) reduced by the array gain (AG) of the directional receiver,

$$\boxed{DNL = NL - AG}$$

where DNL = unwanted noise detected by the directional receiver of bandwidth w.

NL = Noise Level in the environment (as would be detected by an omnidirectional receiver of bandwidth w).

AG = Array Gain, the improvement (reduction) in DNL resulting from the array design and processing.

RECALL:

If the noise is not directional, and if the signal is an ideal plane wave (or at least close to it), then AG and DI are essentially the same.

NOISE LEVEL

$$DNL = NL - AG$$

$$\boxed{NL = NSL + 10 \log w}$$

where

NL = noise level, the acoustic energy in dB found in the bandwidth of the system,

NSL = noise spectrum level, the pressure spectrum level of the unwanted noise,

and

w = the bandwidth of the system in Hz.

NOTE: THE SAME LIMITATIONS APPLY IN USING THIS FORMULA AS WERE POINTED OUT IN THE DISCUSSION OF THE BROADBAND SL.

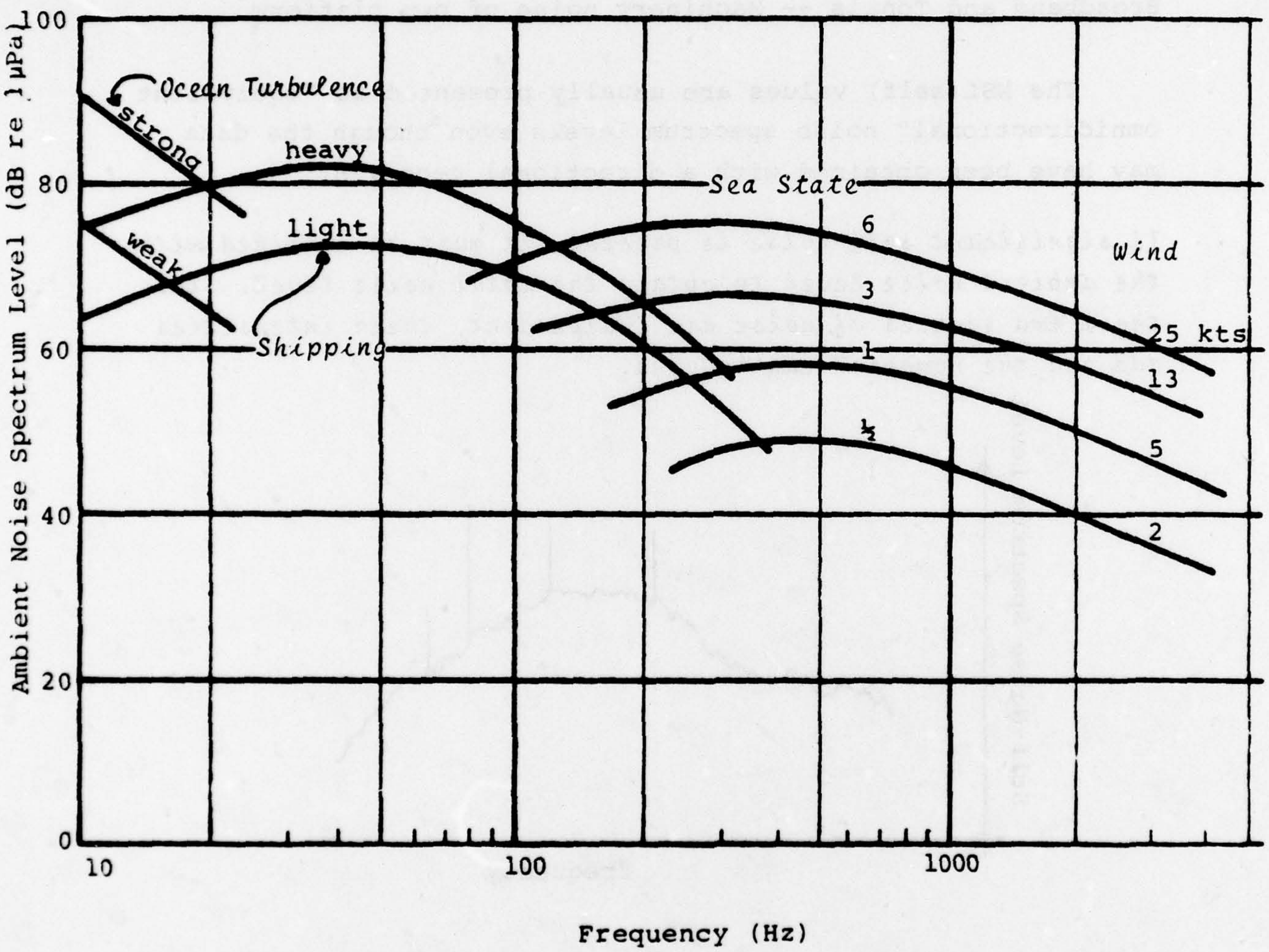
There are two basic sources of noise in SONAR:

1. **AMBIENT NOISE** -- *noise existing in the environment in the absence of both the receiving platform and the target.*
2. **SELF NOISE** -- *noise created by the receiving platform.*

If both ambient- and self-noise are competitive, then the combined noise level is found by combining NL (ambient) and NL (self) with the help of the nomogram.

AMBIENT-NOISE SPECTRUM LEVEL (Deep water)

NL(ambient) = NSL(ambient) + 10 ℓog w



Adapted from: Wenz, Jour. Acoust. Soc. Am. 34, 1936 - 1956 (1962)
Perrone, ibid 46, 762 - 770 (1969)

SELF-NOISE SPECTRUM LEVEL

$$NL(self) = NSL(self) + 10 \log w$$

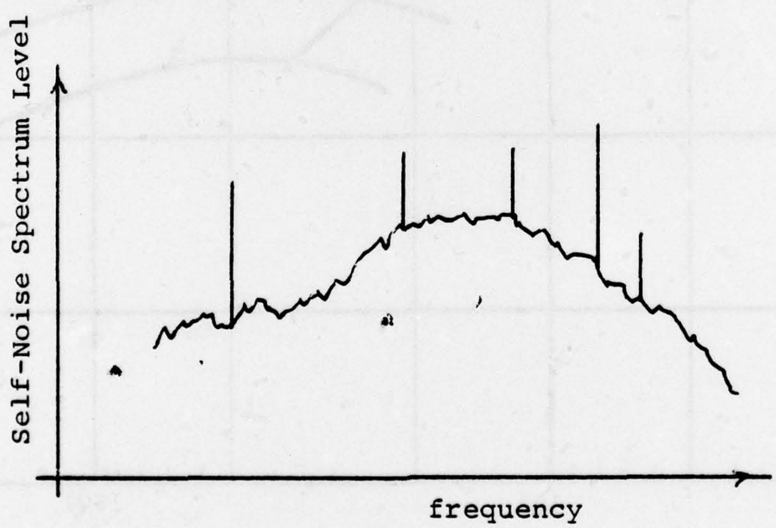
Sources of Self-Noise:

Broadband -- Hydrodynamic noise due to flow past receiver/acceleration of receiver

Broadband and Tonals -- Machinery noise of own platform

The NSL(self) values are usually presented as "equivalent omnidirectional" noise spectrum levels even though the data may have been obtained with a directional receiver.

If significant self-noise is present, it must be combined with the ambient noise level to obtain the total noise level. Since these two sources of noise are independent, their intensities add and the nomogram can be used.



REVERBERATION (ACTIVE SONAR)

$$DNL = RL$$

If the sound field generated by the source is strong enough, and if there are enough scatterers present in the vicinity of the desired target, then the scattered sound from the undesired scatterers is received along with the echo from the desired target. This undesired sound is called <u>reverberation</u>.

The echo level of the signal received from the desired target is

$$EL = SL - 2TL + TS \quad \text{(monostatic)}$$

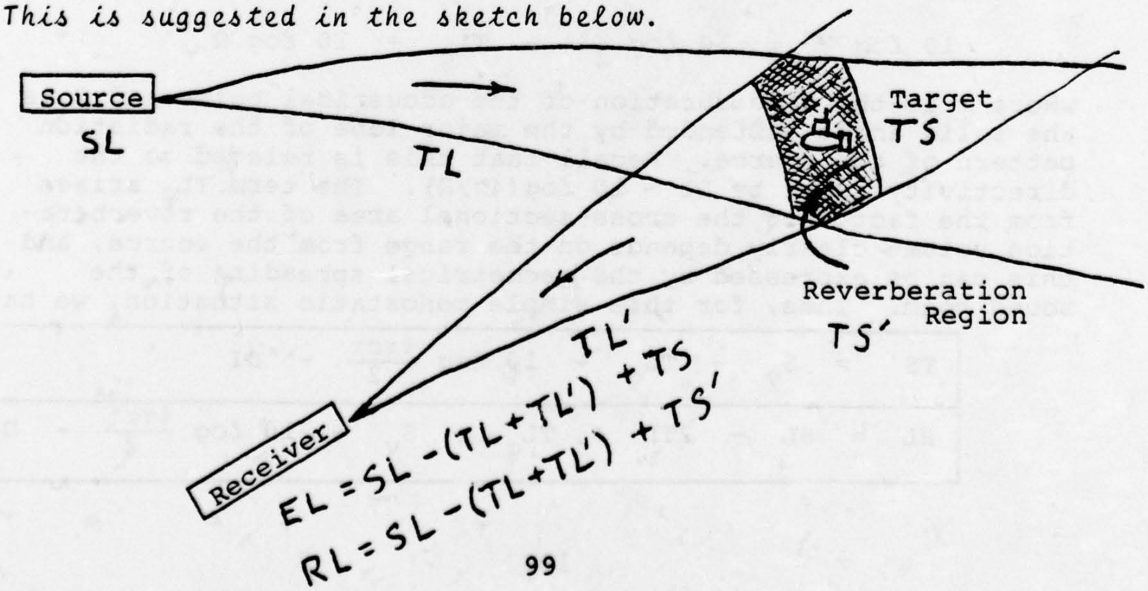
where TS is the target strength of the desired target. The <u>reverberation level</u> RL can be written in the form

$$RL = SL - 2TL + TS' \quad \text{(monostatic)}$$

where TS' is the combined target strength of all the undesired scatterers. When reverberation is the dominant source of unwanted sound, the SONAR equation becomes

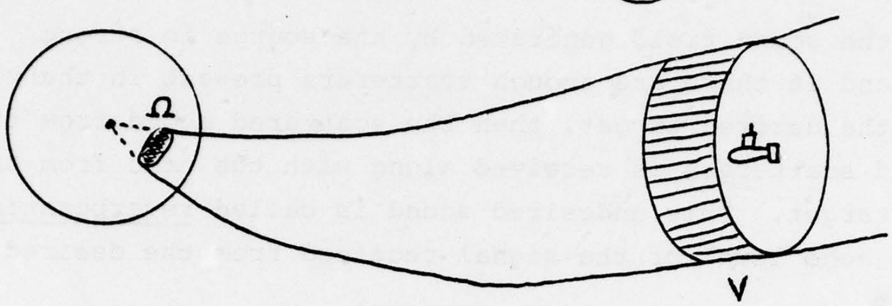
$$\boxed{\begin{array}{c} EL \geq RL + DT \\ \text{or} \\ TS \geq TS' + DT \end{array}}$$

The central problem in obtaining TS' is to determine the volume or area which scatters the sound so that it arrives at the receiver at the same time as the signal from the desired target. This is suggested in the sketch below.



VOLUME REVERBERATION

$$RL = SL - 2TL + TS'$$



The target strength TS' of the undesired scatterers can be written as

$$TS' = 10 \log s_v V$$

where V is the volume from which the reverberation sent to the receiver is in competition with the reflected signal from the desired target and s_v is a measure of the scattering cross section per unit volume. It has become conventional to express s_v in dB terms through the definition

$$S_v = 10 \log s_v$$

The calculation of the reverberation volume V lies at the heart of the problem. This is a study in geometry, involving the way in which major lobes of source and receiver overlap.

If we restrict ourselves to the monostatic situation and assume that the major lobe of the receiver completely overlaps the major lobe of the source, it then follows that the volume V can be written

$$10 \log V = 10 \log \frac{c\tau}{2} + TL_g + 10 \log \Omega$$

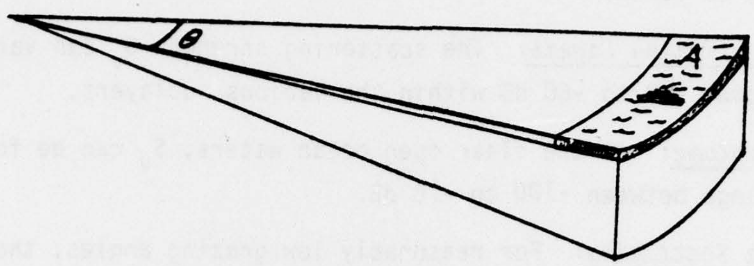
where τ is the time duration of the acoustical pulse and Ω is the solid angle subtended by the major lobe of the radiation pattern of the source. Recall that this is related to the directivity index by $DI \sim 10 \log(4\pi/\Omega)$. The term TL_g arises from the fact that the cross-sectional area of the reverberation volume clearly depends on the range from the source, and this can be expressed by the geometrical spreading of the sound beam. Thus, for this simple monostatic situation, we have

$$\boxed{TS' = S_v + TL_g + 10 \log \frac{4\pi c\tau}{2} - DI}$$

$$\boxed{RL = SL - 2TL + TL_g + S_v + 10 \log \frac{4\pi c\tau}{2} - DI}$$

SURFACE REVERBERATION

$$RL = SL - 2TL + TS'$$



In the case of scattering which arises from a surface (such as the sea surface or the bottom) or from some well-defined layer such as the bubble layer underlying the water-air interface, the reverberation target strength is expressed as

$$TS' = S_s + 10 \log A$$

where A is the surface area generating reverberation in competition with the desired target and S_s is the <u>scattering strength</u> (per unit area).

As before, the problem is one primarily of geometry, involving the way in which source and receiver major lobes overlap. If we again restrict ourselves to the simple monostatic geometry used previously (for which the major lobe of the receiver completely overlaps that of the source), then the surface area of interest can be seen to be

$$A = \tfrac{1}{2} r \, \theta \, c\tau$$

where θ is the angle <u>in radians</u> of the horizontal angular width of the radiation pattern of the source. This then gives the equations

$$\boxed{TS' = S_s + 10 \log r + 10 \log \frac{\theta c\tau}{2}}$$

$$\boxed{RL = SL - 2TL + 10 \log r + S_s + 10 \log \frac{\theta c\tau}{2}}$$

REPRESENTATIVE SCATTERING STRENGTHS

The values of scattering strengths quoted below are representative only. Large deviations can occur depending on location, season, biological activity, and so forth.

Deep Scattering Layers: The scattering strength S_v can vary between about -90 to -60 dB within the various sublayers.

Water Volume: In the clear open ocean waters, S_v can be found to range between -100 to -70 dB.

Surface Scattering: For reasonably low grazing angles, the combined scattering from the ocean surface and the population of bubbles found at shallow depths below the surface gives scattering strengths S_s ranging around -50 to -30 dB for sea states between 1 and 4.

Bottom Scattering: For grazing angles between 20^o and 60^o, representative values of the scattering strength S_s range between -40 and -10 dB. The exact values depend very strongly on the particular bottom and its composition. In general, however, as the grazing angle goes to zero, the scattering becomes negligible and S_s goes to $-\infty$ dB.

REVERBERATION LEVEL DEPENDENCIES

Notice that for either volume or surface reverberation the reverberation target strength TS' increases with increasing range, either as TL_g (volume) or 10 ℓog r (surface). This is to be compared with ambient noise masking, for which there is no range dependence. *This means that, given sufficient source level, as SL increases SONAR performance will change from ambient-noise-limited performance to reverberation-limited performance.* While detection for ambient-noise limited performance depends on the source level and directivity index, notice that when the performance becomes reverberation limited there is no dependence on the source level. *Additional power will not improve reverberation-limited performance.* Reverberation can be reduced by increasing the directivity of the source and/or receiver, or by decreasing the pulse length, given by $c\tau$. Increased directivity has the disadvantage of reducing scanning rates, and decreased pulse length may result in reduction of the target strength TS of the desired target.

TOTAL COMBINED DNL

Self Noise (Active and Passive)

$$DNL_S = NSL_S + 10 \log w - AG$$

Ambient Noise (Active and Passive)

$$DNL_A = NSL_A + 10 \log w - AG$$

Reverberation (Active)

$$DNL_R = RL = SL - 2TL + TS'$$

Passive SONAR:

DNL_S and DNL_A must be combined with the help of the nomogram to obtain the total Detected Noise Level DNL_T.

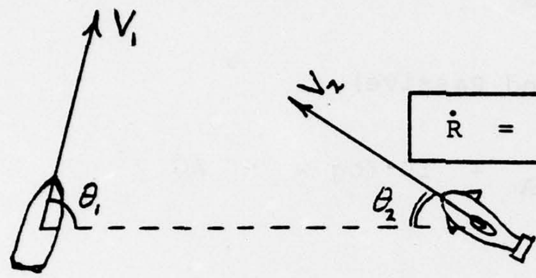
Active SONAR:

(1) If the frequency of the echo cannot be distinguished from that of the reverberation, then DNL_R, DNL_A, and DNL_S must be combined.

(2) If the frequency of the echo can be distinguished from that of the reverberation, then DNL_A and DNL_S must be combined as in passive SONAR.

RANGE RATE

The frequency of the signal obtained by the receiver depends not only on the frequency transmitted but also on the relative motion between source and receiver. This relative speed is characterized by a Range Rate defined as



$$\boxed{\dot{R} = \left|V_1 \cos\theta_1 + V_2 \cos\theta_2\right|},$$

where

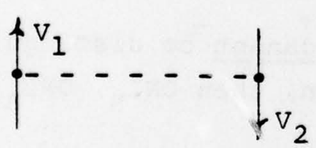
$\vec{V}_1$ = velocity of vessel 1; its speed is $V_1 = |\vec{V}_1|$

$\vec{V}_2$ = velocity of vessel 2; its speed is $V_2 = |\vec{V}_2|$

θ_1 = angle between line of centers and velocity of vessel 1

θ_2 = angle between line of centers and velocity of vessel 2

SOME EXAMPLES

1. 

$\dot{R} = 0$

2. 

$\dot{R} = V_1 + V_2$

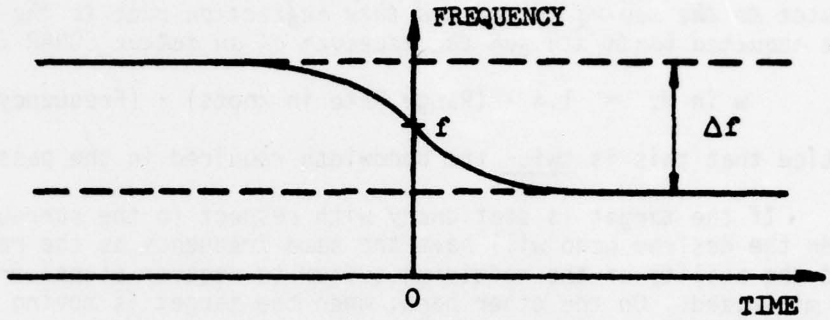
3. 

$\dot{R} = |V_1 - V_2|$

4. 

$\dot{R} = V_1$

DOPPLER SHIFT FOR PASSIVE SONAR



For t << 0, receiver hears frequency $f + \frac{1}{2}\Delta f$ (range closing)

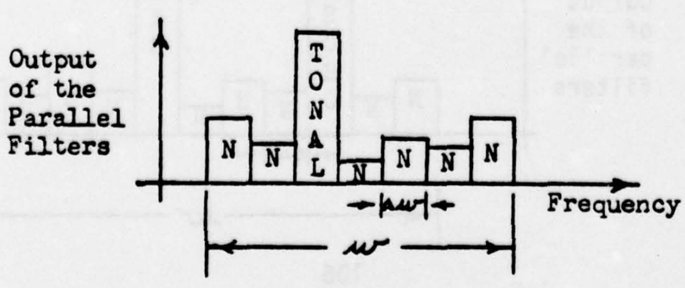
For t >> 0, receiver hears frequency $f - \frac{1}{2}\Delta f$ (range opening)

The necessary bandwidth w *of the receiver must be great enough to accomodate the maximum expected Doppler shifts.* While decreasing w will improve the ability of a passive SONAR to detect tonals, it cannot be made smaller than the expected Doppler shifts. *For a passive SONAR system, this requires that the minimum acceptable bandwidth of the entire system be*

$$w \text{ in Hz} = 0.7 \cdot (\text{range rate in knots}) \cdot (\text{frequency in kHz})$$

Range Rate is the speed with which two objects approach or recede from each other; it is the relative speed.

If a tonal of known frequency f is being received, and if it is desired to determine the range rate, then the bandwidth w can be divided up into smaller intervals Δw by the use of parallel filters. The output of each filter is processed, and the particular filter which provides a detection indicates, within Δw, the Doppler shift of the detected tonal and thus the range rate.



DOPPLER-SHIFT FOR ACTIVE SONAR

In the case of active SONAR, any motion of the platform will cause the generated signal propagated into the water to have a frequency shift which depends on the bearing into which it is sent. This Doppler-shifted signal is subjected to a further Doppler-shift when it is reflected from the moving target. Without going into the specifics of the various frequency shifts, we shall simply quote some results of interest:

Because of the two-way propagation of acoustical energy from the moving source to the moving target and then reflection back to the moving receiver, the required bandwidth for the receiver of an active SONAR is

$$w \text{ in Hz} = 1.4 \cdot (\text{Range Rate in knots}) \cdot (\text{Frequency in kHz})$$

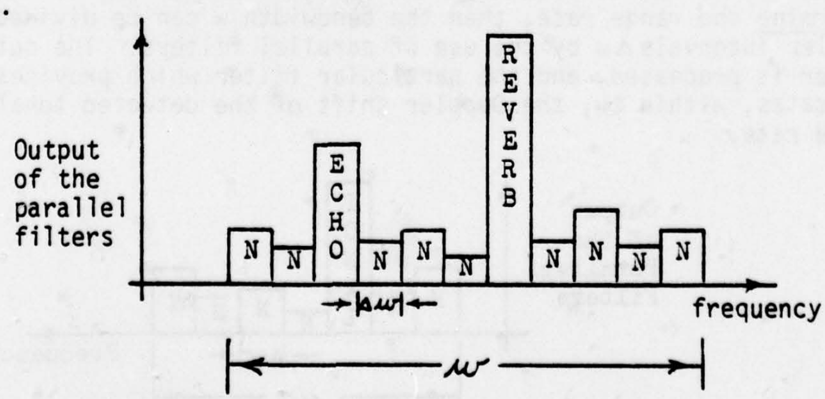
Notice that this is <u>twice</u> the bandwidth required in the passive SONAR case.

If the target is stationary with respect to the surrounding water, then the desired echo will have the same frequency as the reverberation and the ability of the receiving system to recover signal from noise will be minimized. On the other hand, when the target is moving through the water, then the echo and reverberation will have different frequencies and detection may be much easier.

(This effect is easy to perceive: For example, if you listen to the received echo and the accompanying reverberation, it is quite difficult to be sure that a weak echo is present if there is no Doppler-shift with respect to the reverberation. On the other hand, if there is a Doppler-shift, then the change in frequency heard when the echo arrives is quite noticeable even for a rather weak echo.)

These considerations point to the use of parallel filters of relatively narrow bandwidths to facilitate the separation of signal from reverberation when the difference in frequency is small.

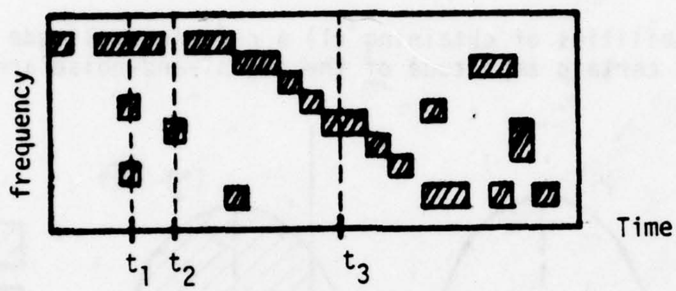
The use of parallel filters in the receiver of an active system results in noise-limited performance in all frequency intervals Δw *which do not encompass the reverberation frequency.* If the echo from the moving target is shifted into one of these intervals, detection is noise-limited. If the target is stationary, or sufficiently slow, then the echo appears in the same frequency interval as the reverberation, and detection is reverberation limited if the reverberation is stronger than the ambient noise in that interval.



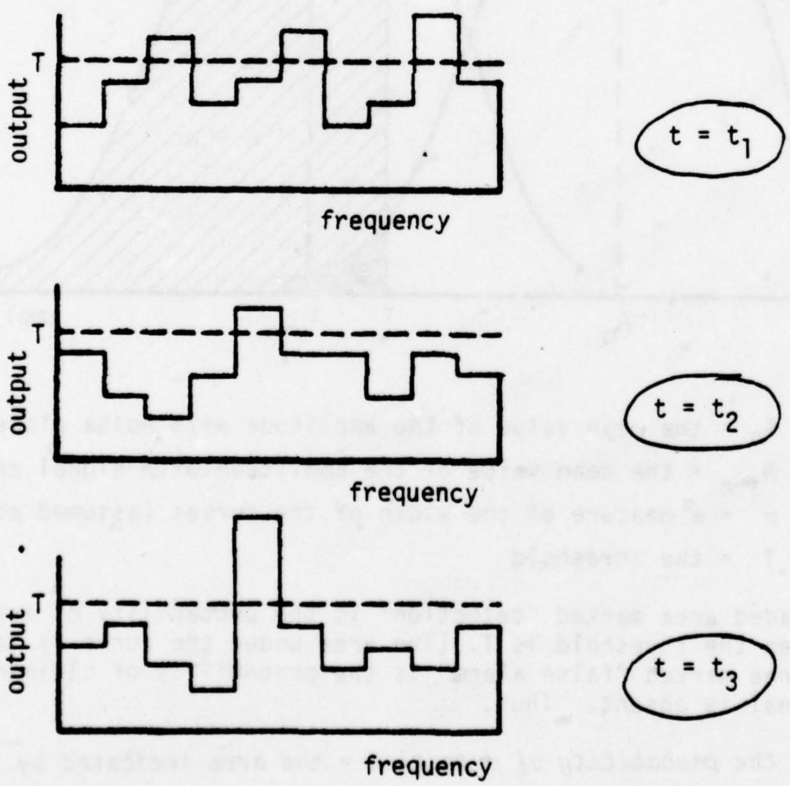
DETECTION

To take advantage of a narrow bandwidth and still permit reception of a Doppler-shifted signal, it is necessary to design a system that employs many parallel filters (each of narrow bandwidth Δw) which together cover the entire frequency range of interest.

The output of such a system might be presented in the manner shown below, where the shaded areas represent individual filter outputs which have exceeded the threshold T:



At any instant the outputs of the various filters might look like



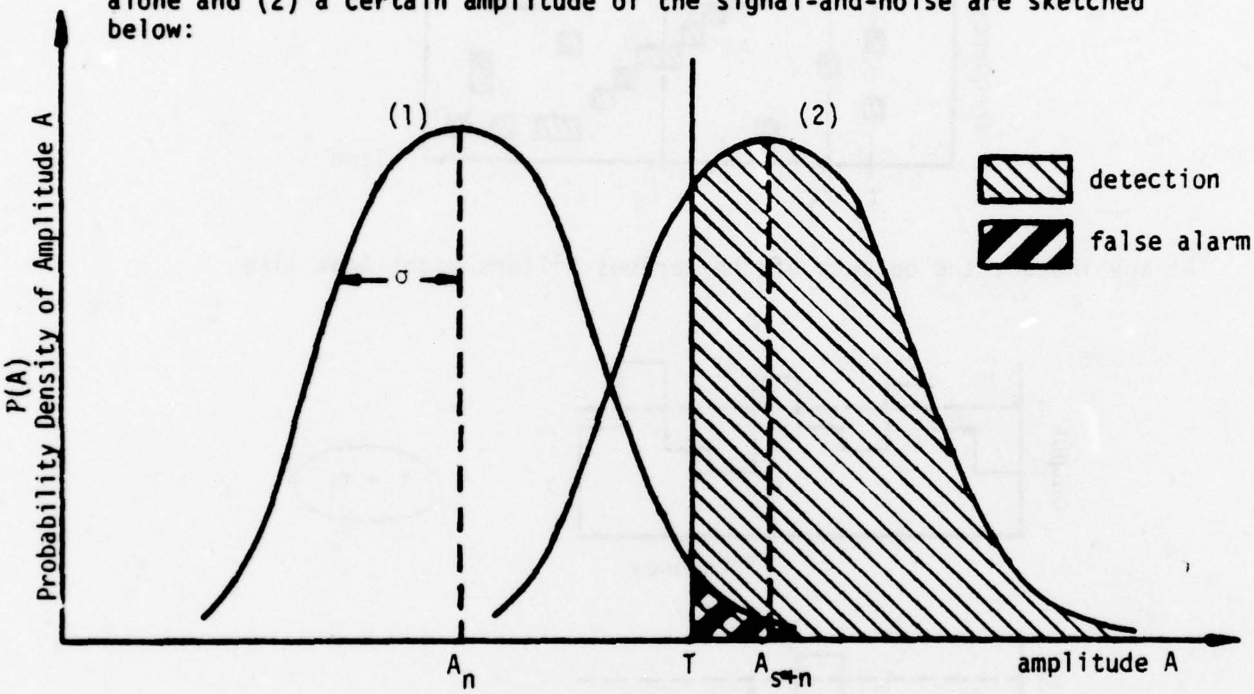
where T is the threshold setting.

DETECTION THRESHOLD

$$EL \geq DNL + DT$$

The detection process consists of designating a threshold T which, when exceeded, causes a detection to be recorded. If the signal is much stronger than the noise, it is clear that a threshold can be defined that will allow valid signals to be recorded while ignoring the noise. However, when the signal and noise are of comparable intensity, any threshold that will catch a reasonable number of valid signals will also record "detections" when a valid signal is absent.

The probabilities of obtaining (1) a certain amplitude of the noise alone and (2) a certain amplitude of the signal-and-noise are sketched below:



where A_n = the mean value of the amplitude with noise alone

A_{s+n} = the mean value of the amplitude with signal and noise

σ = a measure of the width of the curves (assumed equal)

T = the threshold

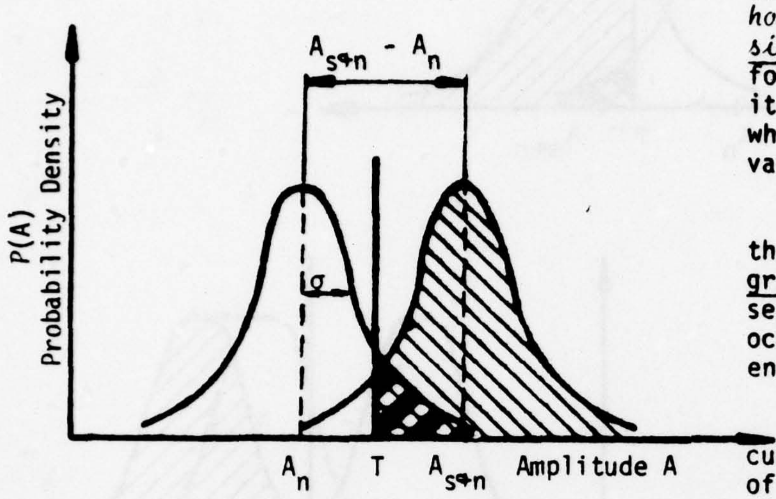
The shaded area marked "detection" is the probability of making a valid detection when the threshold is T. (The area under the curve is taken as unity.) The shaded area marked "false alarm" is the probability of claiming a detection when the signal is absent. Thus,

P(D) = *the probability of detection* = the area indicated by

P(FA) = *the probability of a false alarm* = the area indicated by

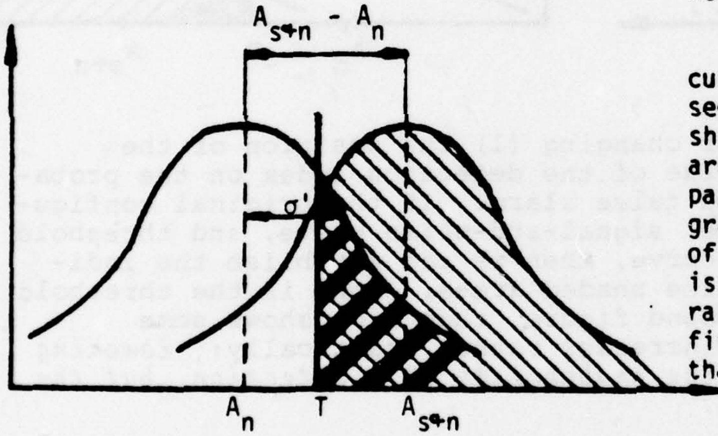
DETECTION INDEX d

We need to find some simple criterion d which will describe how the curves of noise and signal-and-noise overlap, since for a given value of threshold, it is the overlap of the curves which establishes the relative values of P(D) and P(FA).



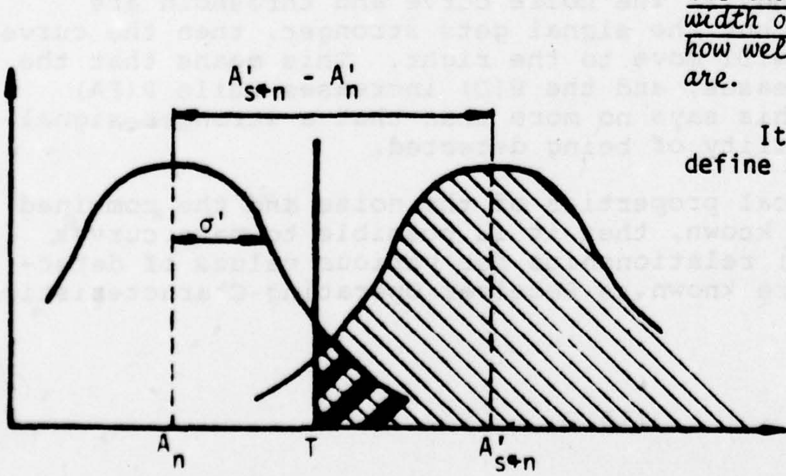
Notice the configuration of the two curves given in the first graph. The curves are quite well-separated, in that their peaks occur at amplitudes whose difference is larger than the σ.

In the second graph, each curve has the same average value of A as before, but now the $\sigma' = 2\sigma$. The curves now are not well-separated.



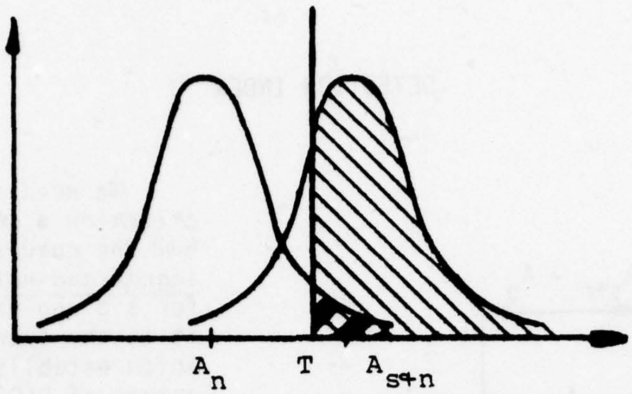
In the third graph, the curves have the same σ' as in the second, but the two have been shifted farther apart so that they are well-separated. Indeed, comparison of the first and third graphs will reveal that the amount of overlap of the pairs of curves is the same: Notice that the ratio of A_{s+n} - A, to σ in the first graph is identical with that for the third.

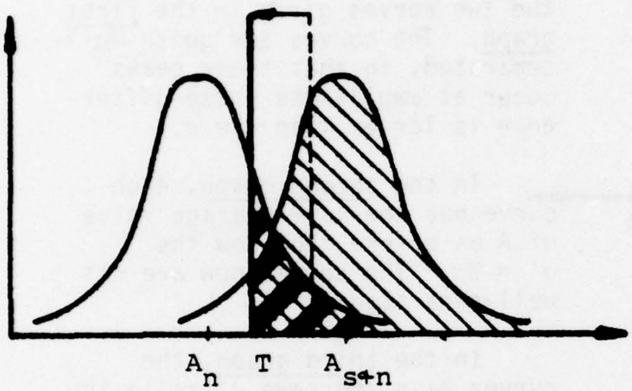
It is the separation between the two curves in units of the width of the curves which measures how well-separated the two curves are.

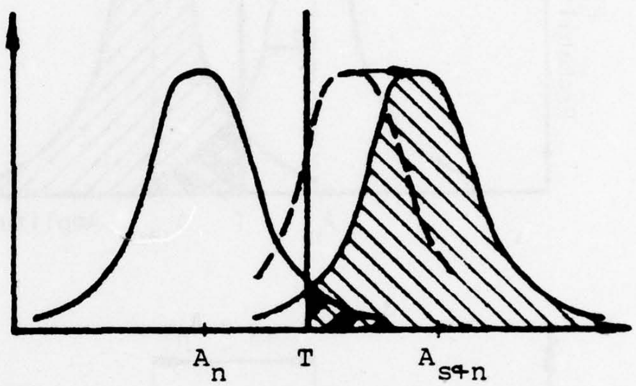


It is thus plausible to define the detection index d by

$$d = \left(\frac{A_{s+n} - A_n}{\sigma} \right)^2$$







Study the effects of changing (1) the position of the threshold and (2) the value of the detection index on the probabilities of detection and false alarm. If the original configuration of the noise curve, signal-and-noise curve, and threshold are as given in the top curve, then we can establish the indicated P(D) and P(FA) by the shaded areas. Now, if the threshold is lowered, as in the second figure, then P(D) shows some increase, but the P(FA) increases rather drastically: *lowering the threshold increases the probability of a detection, but the "clutter" also increases.*

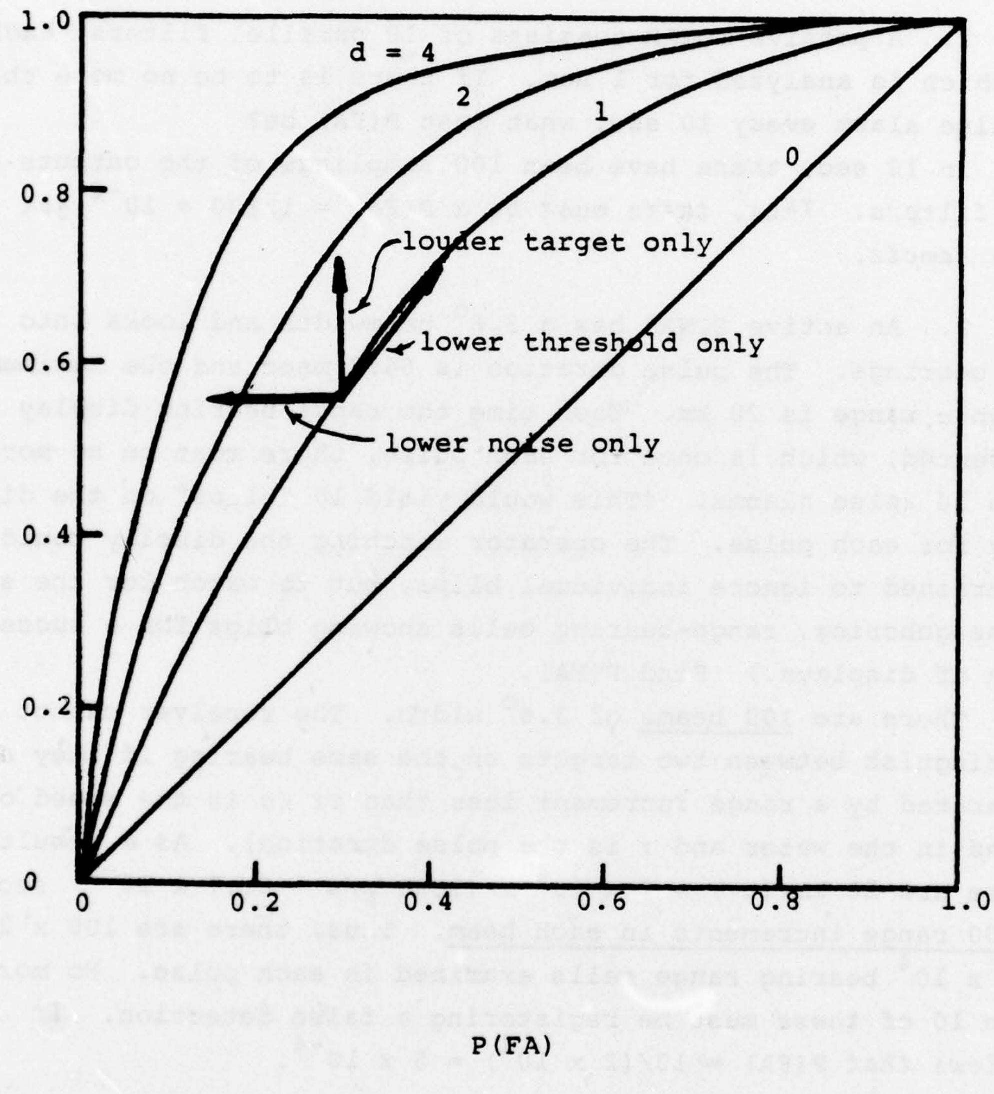
On the other hand, if the noise curve and threshold are maintained constant, but the signal gets stronger, then the curve of signal-and-noise will move to the right. This means that the detection index increases, and the P(D) increases while P(FA) remains the same. This says no more than that a stronger signal has a greater probability of being detected.

If the statistical properties of the noise and the combined signal-and-noise are known, then it is possible to make curves of the P(D) and P(FA) relationships for various values of detection index. These are known as Receiver Operating Characteristics (ROC) curves.

RECEIVER OPERATING CHARACTERISTICS

ROC

A representative set of ROC curves is shown below:



Each SONAR *receiver will have its own individual set of* ROC *curves.*

DETERMINING THE REQUIRED P(FA)

To avoid disruptive numbers of false alarms, the probability of obtaining a false alarm from each analysis of the received signal must be carefully controlled:

EXAMPLES:

1. A passive SONAR consists of 10 parallel filters, each of which is analyzed for 1 sec. If there is to be no more than 1 false alarm every 10 sec, what must P(FA) be?

In 10 sec, there have been 100 samplings of the outputs of the filters. *Thus, there must be a* P(FA) = 1/100 = 10^{-2} *for each sample.*

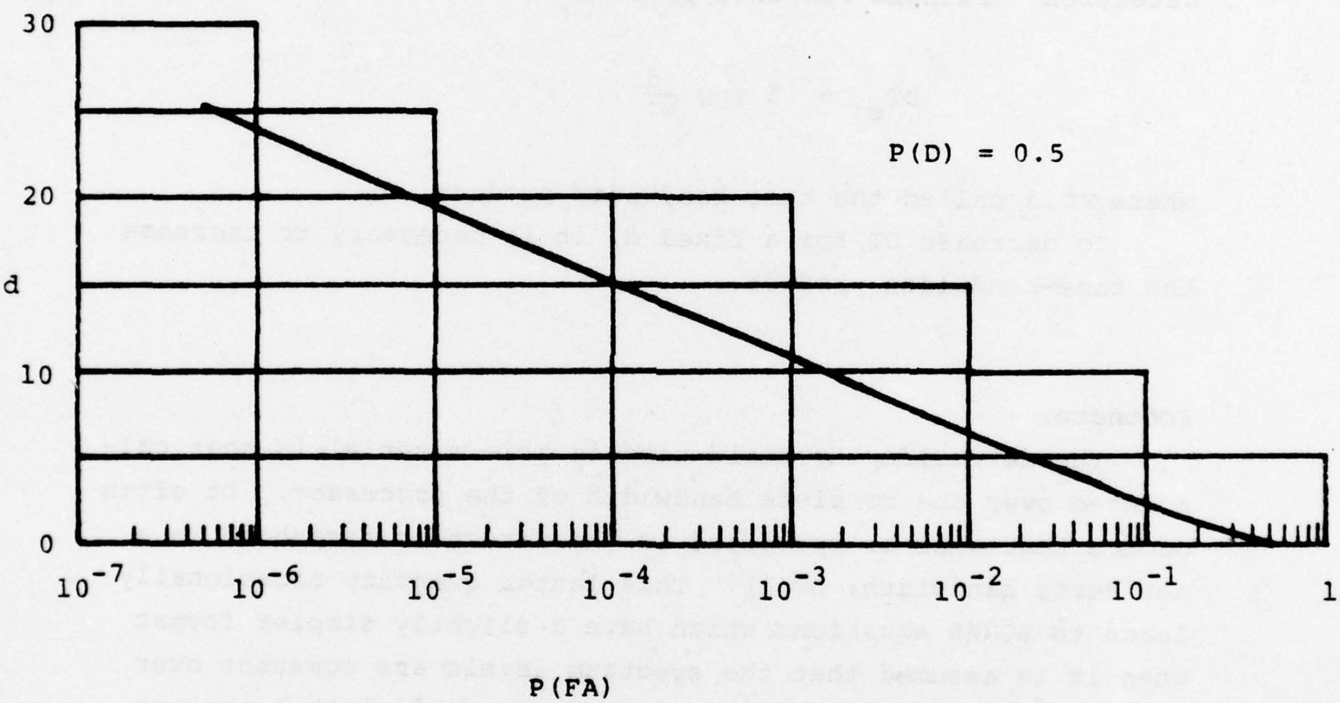
2. An active SONAR has a 3.6° beamwidth and looks into all bearings. The pulse duration is 66.7 msec and the maximum useable range is 20 km. Each time the range-bearing display is presented, which is once for each pulse, there must be no more than 10 false alarms. (This would yield 10 "blips" on the display for each pulse. The operator watching the display could be trained to ignore individual blips, but to watch for the same, or neighboring, range-bearing cells showing blips for a succession of displays.) Find P(FA).

There are 100 beams of 3.6° width. The receiver cannot distinguish between two targets on the same bearing if they are separated by a range increment less than $c\tau$ (c is the speed of sound in the water and τ is the pulse duration). As a result, there are 20 km/($c\tau$) = 2×10^4 m/(1500 m/s $\cdot$ 66.7×10^{-3} sec) = 200 range increments in each beam. Thus, there are 100 x 200 = 2×10^4 bearing range cells examined in each pulse. No more than 10 of these must be registering a false detection. *It follows that* P(FA) = $10/(2 \times 10^4) = 5 \times 10^{-4}$.

Notice that d must be large enough that for P(D) = 0.5 the P(FA) will not exceed the maximum decided upon. This sets a lower limit on d and an upper limit on the range of detection.

DETECTION INDEX AND P(FA) FOR SPECIFIED P(D)

The curve below shows a *REPRESENTATIVE* plot of the relationship between d and P(FA) for the case P(D) = 0.5. It was obtained from a set of ROC curves similar to those on page 111. A similar curve could be obtained for any specified P(D).



This curve is representative only! Every SONAR system will have its own specific ROC curves and therefore its own set of d vs. P(FA) curves for each P(D).

DETECTION THRESHOLD (PASSIVE)

$$SL - TL \geq DNL + DT$$

For a passive detection system with a completely unknown signal and Gaussian noise,* the best performance is obtained by filtering in a bandwidth w and then taking the square of the signal, and accumulating over a processing time τ. If the signal-to-noise ratio is small, it can be shown that the detection threshold for this process is

$$DT_e = 5 \log \frac{d}{w\tau}$$

where $w\tau$ is called the time-bandwidth product.

To decrease DT for a fixed d, it is necessary to increase the time-bandwidth product.

Footnote:

The detection threshold used in this material is that calculated over the complete bandwidth of the processor. It often occurs that what is specified is the detection threshold in a one Hertz bandwidth, DT(1). This latter quantity occasionally leads to SONAR equations which have a slightly simpler format when it is assumed that the spectrum levels are constant over the bandwidth of the system. Otherwise, we feel that the use of DT(1) introduces more possibilities for confusion, so that its use creates more problems than it solves. [*In particular, if Urick's book is studied, it must be fully understood that whereas he uses* DT *throughout most of the text, his chapter devoted to the detection threshold uses* DT(1).] The formula relating the two quantities is

$$DT(1) = DT + 10 \log w.$$

---

*Gaussian noise means noise whose curve of probability density $P(A)$ vs. amplitude A resembles the "bell-shaped" or Gaussian distribution.

DETECTION THRESHOLD (ACTIVE)

$$SL - 2TL + TS \geq DNL + DT$$

Energy Detection

If, as was the case for the passive SONAR equations, the mechanism for the detection of the desired signal is based on detecting an excess of acoustical energy in the frequency band of interest, then we have the same expression for the detection threshold,

$$DT_e = 5 \log \frac{d}{w\tau}$$

where τ is the integration time. In the case of active sonar, τ should be the duration of the received echo. If the integration time is less than or greater than this pulse duration, it can be shown that the DT is increased above this expression, and performance degraded.

Correlation Detection

An alternative mode of signal processing is possible in the case of active SONAR. Since the amplitude and frequency properties of the tone burst generated by the source are known, it is possible to search for a signal of these same properties in the received echo. If the detailed shape of the received echo matches that of the sent pulse, then it can be shown that the detection threshold is given by

$$DT_c = 10 \log \frac{d}{2w\tau}$$

There are many modifications of this basic idea, but all rely on the technique of multiplying the received signal by a time-delayed model of the sent pulse and integrating the resultant product over the pulse duration. This is done for monotonically increasing delay times, and the resulting function of delay time studied for the presence of a sharp peak representing the matching of the delayed replica with the same signal appearing in the received signal and noise. *In practice, the finite size of the target, multipath interference, and the fluctuations introduced by the inhomogeneities of the transmission of the signal through the water alter the received echo so that the correlation process is not as good as would be desired. As a result, the* DT *for a real system operating in the real world is more than the theoretical expression would predict.*

PUTTING IT ALL TOGETHER

LIST OF SYMBOLS

EL	=	Echo Level (active), or received Signal Level (passive)-- The desired signal level sensed by the detector.
ML	=	Masking Level
DNL	=	Detected Noise Level
DT	=	Detection Threshold
SL	=	Source Level
TL	=	Transmission Loss
TS	=	Target Strength of the desired target (active)
BL	=	Band Level
PSL	=	Pressure Spectrum Level
SSL	=	Source Spectrum Level
NL	=	Noise Level
NSL	=	Noise Spectrum Level
DI	=	Directivity Index of acoustical source or receiver
AG	=	Array Gain of the receiver
RL	=	Reverberation Level (active)
TS'	=	Target Strength of scatterers causing reverberation (active)
τ	=	pulse duration of the acoustical signal or processing time in the receiver
s_v	=	a measure of the scattering cross-section per unit volume
S_v	=	scattering strength for volume scatterers
s_s	=	a measure of the scattering cross-section per unit area
S_s	=	scattering strength for surface scatterers
w	=	bandwidth of the receiver
d	=	detection index for the receiver

PASSIVE SONAR EQUATIONS

$$\boxed{SL - TL \geq DNL + DT}$$

$$DNL = NSL + 10 \log w - AG$$

BROADBAND WITH NEGLIGIBLE TONALS

If SSL and NSL are essentially constant over the band width w of the receiver,

$$\boxed{(SSL + 10 \log w) - TL \geq [(NSL + 10 \log w) - AG] + DT}$$

or, as is often written,

$$SSL - TL \geq NSL - AG + DT$$

TONALS WITH NEGLIGIBLE CONTRIBUTION FROM BACKGROUND

If SL is the source level of the tonal(s) in the bandwidth w of the receiver and if NSL is essentially constant over w, then

$$\boxed{SL - TL \geq [(NSL + 10 \log w) - AG] + DT}$$

If <u>both</u> tonals and broadband noise appear in the receiver bandwidth, the total Source Level must be obtained by subdividing the full bandwidth into smaller segments, finding the Band Level for each segment and then combining all the levels.

ACTIVE SONAR EQUATIONS

$$\boxed{SL - 2TL + TS \geq DNL + DT}$$

$$DNL = \begin{cases} NSL + 10 \log w - AG & \text{for noise} \\ RL & \text{for reverberation} \end{cases}$$

NOISE-LIMITED CONDITIONS

$$\boxed{SL - 2TL + TS \geq NSL + 10 \log w - AG + DT}$$

REVERBERATION-LIMITED CONDITIONS

$$\boxed{SL - 2TL + TS \geq RL + DT}$$

or

$$TS - TS' \geq DT$$

EXAMPLE 1

A submarine with a 160 dB re 1μPa line at 500 Hz crosses a convergence zone at 40 km from an omnidirectional hydrophone. The bandwidth of the receiver is sufficient to encompass a range rate of 20 kts and the Sea State is 3. Assuming a convergence gain of 11 dB, find out how long the signal must be accumulated to give a P(D) = 0.50 and a P(FA) = 10^{-4}.

Solution:

The passive SONAR Equation is

$$SL - TL \geq DNL + DT .$$

Given:

$SL = 160$ dB re 1μPa

$f = 500$ Hz

$r_{CZ} = 40$ km

$\dot{R} = 20$ kt (required capability)

$DI = 0$ (omnidirectional receiver)

$G = 11$ dB

The expected Transmission Loss at 40 km is:

$$\begin{aligned} TL &= 20 \log r + ar - G \\ &= 20 \log(4 \times 10^4) + (2 \times 10^{-5})(4 \times 10^4) - 11 \text{ dB} \\ &= 92.0 + 0.8 - 11 = 82 \text{ dB} \end{aligned}$$

The Detected Noise Level in a Sea State 3 is found from

$$DNL = NSL + 10 \log w - DI$$

where

$$w = 0.7(20)(0.5) = 7 \text{ Hz}$$

Thus

$$\begin{aligned} DNL &= 66 \text{ dB} + 10 \log 7 - 0 \\ &= 66 + 8.5 = 74.5 \text{ dB re } 1\mu\text{Pa} \end{aligned}$$

Now, solve for DT:

$$\begin{aligned} DT \leq SL - TL - DNL &= 160 - 82 - 74.5 \\ &= 3.5 \text{ dB} \end{aligned}$$

Also,

$$DT = 5 \ell og \frac{d}{w\tau}.$$

But $d = 15$ for $P(D) = 0.5$, $P(FA) = 10^{-4}$, as obtained from the curve on page 113. Then

$$\ell og \frac{d}{w\tau} = \frac{3.5}{5}$$

$$\frac{d}{w\tau} = 10^{.7} = 5.0$$

$$\tau = \frac{d}{5.0w}$$

$$\tau = \frac{3}{7} \text{ sec.}$$

EXAMPLE 2

A conventional submarine at periscope depth is traveling at 4 kts. The Sea State is 3 and the mixed layer depth is 100 m. At what range will the submarine be detected by a 36 m deep hydrophone listening at 1000 Hz if DI = 20 dB and DT = 0 dB? Assume broadband detection.

Solution:

The passive SONAR Equation is

$$SSL - TL \geq NSL - DI + DT .$$

Given:

$$\dot{R} = 4 \text{ kt} \qquad f = 1000 \text{ Hz}$$
$$D = 100 \text{ m} \qquad DI = 20 \text{ dB}$$
$$z_o = 36 \text{ m} \qquad DT = 0 \text{ dB}$$

In the absence of other information, SSL is estimated from the material in the text to be

$$SSL = 120 \text{ dB re } 1\mu\text{Pa}$$

For NSL, take the value at 1000 Hz for Sea State 3 from the graph on page 97, NSL = 62 dB re 1μPa.

Now

$$TL \leq SSL - NSL + DI - DT$$

or

$$TL \leq 120 - 62 + 20 - 0 = 78 \text{ dB}$$

For sound to be trapped in the mixed layer, the frequency f must exceed

$$f_{min} = (2 \times 10^5) D^{-3/2} = 200 \text{ Hz}$$

Since f = 1000 Hz > f_{min}, the sound can be trapped, and we can use TL for the mixed layer,

$$TL = 10 \log r_t + 10 \log r + (a + b/r_s) r ,$$

if $r > r_t$.

Since we don't know the exact depth of the submarine noise source in the mixed layer, we use the receiver depth instead to obtain r_t. The receiver is probably deeper than a conventional submarine at periscope depth, so this choice gives a larger r_t and hence a more conservative estimate of TL. Then

$$r_t = 105D/(D - Z_s)^{1/2} = 105 \times 100/8 = 1310 \text{ m}$$

$$r_s = 840\, D^{1/2} = 840 \times 10 = 8400 \text{ m}$$

From the graphs and equations: $a = 6 \times 10^{-5}$ dB/m

$b = 3.1$ dB/bounce (Schulkin)

Then

$$a + b/r_s = 4.3 \times 10^{-4} \text{ dB/m}$$

so that

$$TL = 10\, \ell og\, 1310 + 10\, \ell og\, r + (4.3 \times 10^{-4})\, r \le 78 \text{ dB.}$$

Re-arrange

$$10\, \ell og\, r + (4.3 \times 10^{-4})r \le 47 \text{ dB} \quad \text{for detection.}$$

Solve this by trial and error. Begin by ignoring absorption (thus over-estimating) and find the initial value of r from $\ell og\, r = 4.7$ or $r = 5 \times 10^4$ m. The solution then proceeds as in the table below:

r	10 ℓog r	(4.3×10^{-4})r	Total
5×10^4 m	47.0	21.5	68.5
2×10^4	43.0	8.6	51.6
1.5×10^4	41.8	6.5	48.2
1.3×10^4	41.1	5.6	46.7
1.34×10^4	41.27	5.76	47.0

Range at detection is $r = 1.34 \times 10^4$ m which <u>is</u> beyond r_t.

EXAMPLE 3

Repeat Example 2 with $f = 100$ Hz. Assume "heavy shipping" as would be encountered in the Mediterranean Sea.

Solution:

We have the same SONAR Equation,

$$SSL - TL \geq NSL - DI + DT.$$

$D = 100$ m

$f_{min} = (2 \times 10^5) D^{-3/2} = 200$ Hz

Then $f = 100$ Hz $< f_{min}$ *No trapping*

$$\left.\begin{array}{lll} \text{Sea State 3,} & NSL_1 = & 61 \text{ dB} \\ \text{Heavy Shipping,} & NSL_2 = & 75.5 \text{ dB} \end{array}\right\} NSL = 75.7 \text{ dB (nomogram)}$$

Absorption, $a = 1.0 \times 10^{-6}$ dB/m

$DI = 20$ dB

$DT = 0$ dB

$SSL = 133$ dB from page 84.

Because there is no trapping, and in the absence of any other alternative, we use spherical spreading:

$$TL = 20 \log r + ar$$

Now, $TL \leq SSL - NSL + DI - DT$

or $TL \leq 133 - 75.7 + 20 - 0 = 77.3$ dB

Must solve $20 \log r + (1 \times 10^{-6}) r = 77.3$

Neglect absorption for first estimate: $r = 7.3 \times 10^3$ m.

r	$20 \log r$	$(1 \times 10^{-6}) r$	Total
7.3×10^7	77.3	7.3×10^{-3}	77.3

Thus, absorption proves negligible and the range at detection is just the first estimate:

$$r = 7.3 \times 10^3 \text{ m}$$

This example points out the importance of always checking to see if any assumptions have been violated by the given parameters.

EXAMPLE 4

An active SONAR operates at 1 kHz with a source level of 220 dB re 1μPa, a directivity index of 20 dB, a horizontal beam width of 10^{o}, and a pulse length of 0.1 sec. Correlation detection is used and it is desired to have P(D) = 0.50 with P(FA) = 10^{-4}. The target strength of the submarine is expected to be 30 dB and its speed may be up to 20 kts. Both the SONAR and the submarine are in a mixed layer of depth 100 m with the source at 36 m. The Sea State is 3. The scattering strength for surface scatterers is -30 dB, and volume scattering is negligible. Find the maximum range of detection for 50% probability of detection.

Solution:

The SONAR Equation is

$$SL - 2TL + TS \geq DNL + DT.$$

Given:	f	= 1 kHz		$\dot{R}$	=	20 kt
	SL	= 220 dB re 1μPa		Sea State	=	3
	τ	= 0.1 sec		S_s	=	-30 dB
	DI	= 20 dB		TS	=	30 dB
	θ	= 10^{o}		D	=	100 m
				Z_s	=	36 m

The statement of the problem suggests that the necessary bandwidth of the receiver is not divided up among parallel processors, so that (1) the ambient noise must be calculated across the full bandwidth of the system, and (2) echo and reverberation cannot be separated in frequency.

Evaluate the separate terms:

$$DT = 10 \log \frac{d}{2w\tau}$$

$d = 15$ by the same calculation as in Example 1.

$$w = 1.4\,\dot{R}\,f = (1.4)(20\text{ kts})(1\text{ kHz}) = 28\text{ Hz}$$

$$DT = 10\,\ell og\,\frac{15}{(2)(28\text{ sec}^{-1})(0.1\text{ sec})} = 4.3\text{ dB}$$

$$TL = 10\,\ell og\,r_t + 10\,\ell og\,r + (a + b/r_s)r$$

$$\left.\begin{aligned} r_t &= 1310\text{ m} \\ r_s &= 8400\text{ m} \\ a + b/r_s &= 4.3 \times 10^{-4} \end{aligned}\right\}\quad \text{from Example 2}$$

$$TL = 10\,\ell og\,r + (4.3 \times 10^{-4})r + 31.2$$

$$DNL_N = NSL - DI + 10\,\ell og\,w \qquad SS = 3$$

$NSL = 62$ dB from graph on page 97

$$DI = 20\text{ dB}$$

$$DNL_N = 62 - 20 + 10\,\ell og\,28 = 42 + 14.5$$

$$= 56.5\text{ dB re }1\mu Pa$$

$$DNL_R = RL = SL - 2TL + TS' \qquad S_s = -30\text{ dB}$$

$$TS = S_s + 10\,\ell og\,\frac{\theta c\tau}{2} + 10\,\ell og\,r \qquad \theta = 10^{\circ}\left(\frac{\pi}{180^{\circ}}\right) = \frac{\pi}{18}$$

$$= -30 + 10\,\ell og\,\frac{(\pi/18)(1500)(0.1)}{2} + 10\,\ell og\,r$$

$$= -30 + 11.2 + 10\,\ell og\,r$$

$$= -18.8 + 10\,\ell og\,r$$

$$DNL_R = 220 - 2[10\,\ell og\,r + (4.3 \times 10^{-4})r + 31.2] + [-18.8 + 10\,\ell og\,r]$$

$$= 138.8 - 10\,\ell og\,r - (8.6 \times 10^{-4})r$$

Rewrite the SONAR Equation

$$2TL \le SL + TS - DNL - DT$$

$$\le 220 + 30 - 4.3 - DNL$$

or $$2TL \le 245.7 - DNL$$

Range	2TL	DNL_R	DNL_N	DNL	FOM = 245.7 - DNL	2TL - FOM
10^4 m	150.9 dB	90.2	56.5	90.2	155.5	-4.6
2×10^4	165.6	78.6	56.5	78.6	167.1	-1.5
4×10^4	188.8	58.4	56.5	60.6	185.1	+3.7
3×10^4	177.7	68.3	56.5	68.6	177.1	+ .6
2.5×10^4	171.8	73.4	56.5	73.5	172.2	- .4
2.8×10^4	175.4	70.3	56.5	70.5	175.2	+ .2
2.7×10^4	174.2	71.3	56.5	71.4	174.3	- .1
2.73×10^4	174.6	71.0	56.5	71.1	174.6	0

Maximum range is 2.73×10^4 m.

APPENDIX: LOGS

Instructions: For each lettered SECTION, first attempt to do the problems. (The answers are on the last page.)

a) If you encounter no difficulties, skip to the next SECTION.

b) If you do encounter difficulties, read through the discussion and then try the problems again. Go to the next SECTION only when you can do the problems without difficulty.

A. EXPONENTIAL NOTATION

Problems A

1. Write each of the following numbers in exponential form:
   a) 2,365,000 = 2.365×10^6
   b) 872
   c) 3340
   d) 42.1
   e) 612
   f) 1.0

2. Write each of the following numbers in conventional form:
   a) 2.689×10^2 = 268.9
   b) 7.30×10^2
   c) 6.29×10^4
   d) 8.9×10^6
   e) 3.261×10^2
   f) 1.00×10^0

3. Write each of the following numbers in exponential form:
   a) 0.00252 = 2.52×10^{-3}
   b) 0.012
   c) 0.00002
   d) 0.01002
   e) 0.00328
   f) 1.0

4. Write each of the following numbers in conventional form:

   a) $1.03 \times 10^{-6} = 0.00000103$

   b) 3.2×10^{-2}

   c) 6.89×10^{-4}

   d) 1.003×10^{-1}

   e) 1.00×10^{-1}

   f) 1.10×10^{-0}

A succinct way of expressing very large and very small numbers is by exponential (or scientific) notation.

For example, one-hundred and twenty-six million can be written either as

$$126{,}000{,}000$$

or as

$$1.26 \times 10^{8},$$

where the exponent 8 denotes the number of places that the decimal must be moved to the right to express the number in its conventional form.

As another example, one-hundred and twenty millionths can be written either as

$$0.000120$$

or as

$$1.2 \times 10^{-4},$$

where the negative exponent denotes that the decimal place is to be moved to the left.

There is no unique way of expressing a number in exponential form. (Note that $1.26 \times 10^{8} = 126 \times 10^{6} = 0.126 \times 10^{9}$.)

A special case that must be watched is

$$\boxed{1 = 1 \times 10^{0} = 10^{0}} \quad .$$

B. ALGEBRA OF EXPONENTS

Problems B

1. Calculate the following and express the answer in exponential form:

   a) $6.15 \times 10^3 + 2.34 \times 10^3 = 8.49 \times 10^3$

   b) $3.2 \times 10^2 - 1.46 \times 10^3$

   c) $9.1 \times 10^4 + 2.4 \times 10^5$

   d) $3.58 \times 10^{-2} + 1.26 \times 10^{-2}$

   e) $6 \times 10^{-6} + 3.281 \times 10^{-4}$

   f) $3.5 \times 10^4 - 2 \times 10^1$

2. Calculate the following and express the answers in exponential form:

   a) $(2.50 \times 10^6) \times (2.00 \times 10^4) = 5.00 \times 10^{10}$

   b) $(4.40 \times 10^4)/(2.20 \times 10^5)$

   c) $(3.26 \times 10^3) \times (2.00 \times 10^{-2})$

   d) $(5.34 \times 10^{-2})/(2.67 \times 10^5)$

   e) $(8.68 \times 10^{-1})/(4.34 \times 10^{-2})$

3. Calculate the following and express the answers in exponential form:

   a) $(1.1 \times 10^4)^2 = 1.21 \times 10^8$

   b) $(2.0 \times 10^{-3})^4$

   c) $(1.44 \times 10^2)^{\frac{1}{2}}$

   d) $(1.44 \times 10^{-2})^{\frac{1}{2}}$

   e) $(8.0 \times 10^6)^{1/3}$

   f) $(2.56 \times 10^4)^{0.5}$

   g) $(8 \times 10^{-6})^{0.3333...}$

To add and subtract numbers expressed in exponential form it is necessary to express both numbers in the same exponent. For example, adding 3.56×10^4 to 2.1×10^3 is accomplished by

$$\begin{array}{r} 3.56 \times 10^4 \\ \underline{0.21 \times 10^4} \\ 3.77 \times 10^4 \end{array}$$

When asked to subtract 8.67×10^{-1} from 2.23×10^3, note that the answer is 2.23×10^3: the term to be subtracted affects the larger term below the figures retained.

Exponential notation is very helpful in calculations involving multiplication and division. We see that

$$120 \times 2000 = 240{,}000$$

or

$$(1.2 \times 10^2) \times (2 \times 10^3) = 2.4 \times 10^5$$

and

$$36{,}000/200 = 180$$

or

$$(3.60 \times 10^4)/(2 \times 10^2) = 1.8 \times 10^2.$$

The rules are simple:

In multiplication, exponents add: $10^n \times 10^m = 10^{(n+m)}$

In division, exponents subtract: $10^n/10^m = 10^{(n-m)}$

A result of these rules is the useful conversion

$$1/10^n = 10^0/10^n = 10^{-n} \ ,$$

which allows exponents to be moved between numerator and denominator.

Power and roots are trivial in exponential notation:

$$(1.2 \times 10^3)^2 = (1.2 \times 10^3) \times (1.2 \times 10^3) = 1.44 \times 10^6,$$

or, for any power n

$$\boxed{(10^m)^n = 10^{mn}}$$

and,

$$(1.44 \times 10^6)^{\frac{1}{2}} = (1.44)^{\frac{1}{2}} \times (10^6)^{\frac{1}{2}} = 1.2 \times 10^3 \ ,$$

or for any power of n

$$\boxed{(10^m)^{1/n} = 10^{m/n}} \ .$$

C. LOGS (IN BASE TEN)

Problems C

1. Evaluate y
   - a) $y = \ell og\ 10^3$
   - b) $y = \ell og\ 10^{-6}$
   - c) $\ell og\ 10^y = 4$
   - d) $\ell og\ 10^y = -3$
   - e) $\ell og\ 10^y = 0$

The log of x is defined as the power to which 10 must be raised to get x:

If

$$10^y = x$$

then

$$\begin{cases} y = \ell og\ x \\ \text{The } \ell og \text{ of } x \text{ is } y \end{cases}$$

The ℓog of 1 is 0: $1 = 10^0$; $\ell og\ 1 = \ell og\ 10^0 = 0$.

The ℓog of 10 is 1: $10 = 10^1$; $\ell og\ 10 = \ell og\ 10^1 = 1$.

The ℓog of 100 is 2: $100 = 10^2$; $\ell og\ 100 = \ell og\ 10^2 = 2$.

Thus,

$$\boxed{\ell og\ 10^y = y}$$

D. DECIMAL EXPONENTS

Problems D

The table below will allow you to calculate, with a fair degree of accuracy in interpolation, all ℓogs.

x	1.0	1.2	1.5	1.7	2.0	2.5	3.0	3.5
$\ell og\ x$	0.00	0.08	0.18	0.23	0.30	0.40	0.48	0.54

x	4.0	5.0	6.0	7.0	8.0	9.0	10.0
$\ell og\ x$	0.60	0.70	0.78	0.85	0.90	0.95	1.00

If more accuracy is desired, it will be necessary to use a slide rule, calculator, or more exact tables.

1. Find
   a) $\ell og\ 4$
   b) $\ell og\ 40$
   c) $\ell og\ 0.4$
   d) $\ell og\ 4^2$
   e) $\ell og\ 1/4$

2. Find the number whose ℓog is:
   a) 0.70
   b) 1.70
   c) -1.30
   d) -1.70
   e) -2.70

3. Find
   a) $\ell og\ 5^{1/3}$
   b) $\ell og\ 0.333...$
   c) $\ell og\ 480$
   d) $\ell og\ 0.08^{1.7}$
   e) $\ell og\ 10^{\pi}$

4. Find the number whose ℓog is:
   a) 3.60
   b) 2.60
   c) -3.10
   d) -2.60
   e) 2.68

So far we have assumed integer exponents, but everything we have said also is true for decimal exponents. For example, since

$$10^{0.30} = 1.995... \doteq 2.0$$

the ℓog of 2.0 is very close to 0.30

$$\ell og\ 2 \doteq 0.30.$$

As another example,

$$\ell og\ 5.0 \doteq 0.70$$

since

$$10^{0.70} \doteq 5.0.$$

Recall that $\ell og\ 10^y = y$. This can be used to generate several very useful relations:

1. Let $x = 10^a$ and $y = 10^b$. Then,

   $$\ell og\ xy = \ell og\ 10^a\ 10^b = \ell og\ 10^{a+b} = a + b = \ell og\ x + \ell og\ y$$

which yields the relation

$$\boxed{\ell og\ xy = \ell og\ x + \ell og\ y}$$

For example, $\ell og\ 36 = \ell og(4 \cdot 9) = \ell og\ 4 + \ell og\ 9 = 0.60 + 0.95 = 1.55.$
As another example,

$$\ell og\ 0.2 = \ell og(2 \times 10^{-1}) = \ell og\ 2 + \ell og\ 10^{-1} = 0.30 - 1.00 = -0.70$$

2. Let $x = 10^a$. Then,

   $$\ell og\ x^n = \ell og\ (10^a)^n = \ell og\ 10^{an} = na = n\ \ell og\ x$$

which yields the relationship

$$\boxed{\ell og\ x^n = n\ \ell og\ x}$$

For example, $\ell og\ 36 = \ell og\ 6^2 = 2\ \ell og\ 6 = 2(0.78) = 1.56$

3. From the above,

   $$\ell og\ \frac{1}{x} = \ell og\ x^{-1} = -1\ \ell og\ x = -\ell og\ x,$$

$$\boxed{\ell og\ \frac{1}{x} = -\ \ell og\ x}$$

For example, $\ell og\ 0.2 = \ell og\ 1/5 = -\ \ell og\ 5 = -0.70$
Any ℓog can be brought into the range of the above table:

For example,

a) To find the ℓog of a number greater than 10, proceed as follows:

$$\ell og\ 7000 = \ell og\ (7 \times 10^3) = \ell og\ 7 + \ell og\ 10^3$$
$$= 0.85 + 3 \qquad = 3.85.$$

b) Analogously, for a number less than 1,

$$\ell og\ 0.003 = \ell og\ (3 \times 10^{-3}) = \ell og\ 3 + \ell og\ 10^{-3}$$
$$= 0.48 - 3 \qquad = -2.52$$

E. THE DECIBEL

Problems E

1. Express these signal-to-noise ratios in dB:
   a) 2
   b) 1/2
   c) 10
   d) 1/10
   e) 100

2. The voltage of a signal is 1 Volt: find the signal level with reference to the following voltages (expressed in Volts):
   a) 10^{-12}
   b) 10^{-6}
   c) 1
   d) 10^{6}
   e) 10^{12}

By convention, when the *log* of a pressure or voltage is multiplied by 20, the resulting number is said to be measured in "decibels", denoted by dB.

$$20 \log 2 = 20\ (0.3) = 6 \text{ dB}$$

$$20 \log 10 = 20\ (1) = 20 \text{ dB}$$

Decibels are used in two different ways:

a) *to express a* ***ratio***, such as the signal-to-noise ratio

$$\boxed{\text{signal-to-noise ratio} = 20 \log \text{(signal voltage/noise voltage)}}$$

b) *to express a* ***level***, such as the signal level

$$\boxed{\text{signal level} = 20 \log \text{(signal voltage/reference voltage)}}$$

In the latter case it is absolutely essential to specify the reference. e.g., "the signal level is 120 dB re 10^{-6} Volts".

ANSWERS

Problems A:

1. a) 2.365×10^{6}
   b) 8.72×10^{2}
   c) 3.34×10^{3}
   d) 4.21×10^{1}
   e) 6.12×10^{2}
   f) 1.0×10^{0}

2. a) 268.9
   b) 730
   c) 62,900
   d) 8,900,000
   e) 326.1
   f) 1.00

3. a) 2.52×10^{-3}
   b) 1.2×10^{-2}
   c) 2×10^{-5}
   d) 1.002×10^{-2}
   e) 3.28×10^{-3}
   f) 10^{0}

4. a) 0.00000103
   b) 0.032
   c) 0.000689
   d) 0.1003
   e) 0.100
   f) 1.10

Problems B:

1. a) 8.49×10^{3}
   b) -1.14×10^{3}
   c) 3.3×10^{5}
   d) 4.84×10^{-2}
   e) 3.34×10^{-4}
   f) 3.5×10^{4}

2. a) 5.00×10^{10}
   b) 2.00×10^{-1}
   c) 6.52×10^{1}
   d) 2.00×10^{-7}
   e) 2.00×10

3. a) 1.2×10^{8}
   b) 1.6×10^{-11}
   c) 1.2×10^{1}
   d) 1.2×10^{-1}
   e) 2.0×10^{2}
   f) 1.6×10^{2}
   g) 2.0×10^{-2}

Problems C:

1. a) 3
   b) -6
   c) 4
   d) -3
   e) 0

Problems D:

1. a) 0.60
   b) 1.60
   c) -0.40
   d) 1.20
   e) -0.60

2. a) 5.0
   b) 50
   c) 0.050
   d) 0.020
   e) 0.0020

3. a) 0.23
   b) -0.48
   c) 2.68
   d) -1.86
   e) 3.14

4. a) 4.0×10^{3}
   b) 4.0×10^{2}
   c) 8×10^{-4}
   d) 2.5×10^{-3}
   e) 4.8×10^{2}

Problems E:

1. a) 6 dB
   b) -6 dB
   c) 20 dB
   d) -20 dB
   e) 40 dB

2. a) 240 dB re 10^{-12} V
   b) 120 dB re 10^{-6} V
   c) 0 dB re 1 V
   d) -120 dB re 10^{+6} V
   e) -240 dB re 10^{+12} V

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